THE
ELEMENTS
OF
OPTICS.

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THE ELEMENTS OF OPTICS.

IN FOUR BOOKS.

Book I. Simple Optics, or Direct Viſion.
Book II. Catoptrics, or Reflected Viſion.
Book III. Dioptrics, or Refracted Viſion.
Book IV. The Conſtruction of Optical Instruments.

Diſcite ô miſeri, et cauſas cognoſcite rerum.
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THE

PREFACE.

THE Ancients, whilſt they made a pretence of ſtudying Philoſophy, and ſearching out the cauſes of natural things, had no other method to proceed by, than forming hypothetical conjectures out of their own heads according to their various fancies, without any foundation in the nature of things. And then they endeavoured to defend their hypotheſes, by plauſible arguments of wit, and the figures of Logic; by which they flattered themſelves, they had fully eſtabliſhed their hypotheſes, and given a true account of natural cauſes and effects, and that beyond the poſſibility of any oppoſition. And all this while they had not the leaſt thought of making ſo much as one experiment, to confirm the truth of their theories; which is the only poſſible way, that any theory in nature can be eſtabliſhed. By this means it came to paſs, that their philoſophy was no more than a philoſophy of words and empty ſounds, conſiſting of endleſs terms of art, without any meaning or ideas annext to them. And whilſt they proceeded in this abſurd method, true philoſophy was quite at a ſtand, and made no manner of progreſs; and the nature of things remained perfectly unknown. Let any operation in nature be propoſed, and they preſently had a name for it; and a few terms of art were to explain its nature, though they could not give us the leaſt notion, how or by what *cauſes ſuch an operation or effect was brought about. And thus they imagined they could ſolve every difficulty in nature, by the mere magic of ſounds. Examples of this way of proceeding are endleſs; but there is nothing in which they have blundered more egregiouſly, than in explaining the nature of light, and the manner of viſion.*

But the moderns finding themſelves no wiſer by this philoſophical gibberiſh, found it neceſſary to purſue another method. They ſaw plainly, that the nature of things could never be known without careful obſervation and experience; therefore they ſet about to make all the experiments upon any ſubject, that could poſſibly be made. By this means they could find out what cauſes could produce ſuch and ſuch effects, what cauſes and effects were linked together, and always accompanied one another; and what proportion they bore to one another; and after they had found theſe things in one ſubject, by applying the ſame method to other ſubjects of the ſame kind; they could find how far theſe laws were general, and agreeable to all of that ſort. In proſecuting this method, it is found, that there are ſome properties that belong but to a few kinds of bodies, and ſome that belong to a great many kinds; and ſome properties that belong univerſally to all bodies: this teaches us ſomething; and the nature and properties of bodies, thus found out by experiments, is called Experimental Philoſophy. *By this method our great* Newton, *the Prince of Philoſophers, found out the nature of light, he analized it, and diſſolved it into its conſtituent particles. He has ſhewn us, that light conſiſts of particles of different colours, and different degrees of refrangibility; and that the colour of each ray of it is permanent and laſting; and its degree of refrangibility invariable: that the particular colour of it can never be changed by any refraction or reflection. But before that, Philoſophers were all of opinion, that light had no difference or variety of its conſtituent*

ſtituent parts, but was in its own nature, ſimple and uniform; and that colours were nothing but certain modifications of the light, or dilatations of the rays, cauſed by reflexions and refractions; whereas they depend upon a real ſeparation of the heterogeneous rays. And all optic writers were generally of opinion, that the reflexion of light was cauſed by the particles thereof ſtriking againſt the ſolid parts of bodies, which they could not enter into, and thence repelled back again, like other reflected bodies. But Sir J. Newton *has ſhewn, that they are driven back and reflected before they touch the body; and that by ſome inherent power in the reflecting body, which acts at a ſmall diſtance from its ſurface, in lines perpendicular thereto.*

Who could ever have thought, that the opacity of bodies depended on their poroſity; and yet the ſame great Philoſopher has ſhewn, that their opacity ariſes from the multiplicity of reflections within the pores of bodies, when the particles are of a determinate ſize. And the ſame bodies become tranſparent, when their pores are filled with a medium of the ſame denſity as the particles of the body.

It is now put paſt diſpute, that viſion is cauſed by the rays of light entering the eye; and painting the image of the object, upon the retina at the bottom of the eye. If theſe rays paſs thro' the ſame medium continued between the object and the eye, then they come from every part of the object to the eye in right lines; then this is called Direct Viſion. *When the rays come not directly to the eye from the object, but fall upon ſome poliſhed body, and are reflected from thence to the eye, in which they form the image of that object; then this ſort of viſion is called* Reflex Viſion. *Here the eye properly ſees the image cauſed by the reflexion, and not the object itſelf. A body muſt be truly poliſhed to give a perfect image by reflection; for a rough uneven ſurface ſcatters the rays, that they make no image, or a very deformed one. If* *the rays paſſing from the object to the eye, go thro' different mediums, as air, water, glaſs, &c. then the rays will be refracted at entering a different medium, and come to the eye in other right lines, of different direction from the former;* and *this is called* Refracted Viſion. *So that there are three different caſes of viſion, whereby objects may be ſeen; and theſe make three diſtinct parts of the ſcience of* Optics; *of which the firſt is the moſt ſimple, and the laſt the moſt complex.*

Since we find the properties of light to agree with the known properties of body, we cannot ſuppoſe light to be any thing elſe but a body, conſiſting of exceeding ſmall particles; which are continually ſhaken off by the vibrating motion of the luminous body, and expelled by the repellent power thereof, with incredible velocity from it in all directions. Some of which, by their exceeding ſmallneſs and tenuity, go thro' every point of ſpace. Some of theſe enter our eyes, whoſe texture and conſtruction is properly adapted thereto, and do there by their motion excite vibrations in the optic nerves, by which we have the ſenſation of ſeeing; and by the wonderful ſtructure of our eyes, give us certain intelligence of objects exiſting without us, and their affections, ſuch as their ſize, form, colour, motion, and even their diſtance: ſo that viſion is not made at random, but is performed by certain uniform and ſteady laws.

It appears by experiment, that light conſiſts of parts both ſucceſſive and contemporary. For one may ſtop that which comes one moment, and let that paſs which comes the next. One may alſo ſtop it in one place, and let it paſs in another. And the part that is ſtopt cannot be the ſame with the part that is gone paſt.

As viſion is the moſt uſeful, as well as the moſt delightful ſenſe we have; ſo that ſcience which teaches the improvement and perfection of this ſenſe, muſt be the moſt valuable of any, and that is Optics, *which not only explains the manner of viſion, but from the* *ſame principles teaches how to ſupply a proper help for bad eyes, and to remedy the ſeveral defects of our ſight, by the help of proper inſtruments contrived according to the rules of this art, ſuch as ſpectacles, reading glaſſes, teleſcopes, microſcopes, &c. and to this art is owing the invention of ſeveral inſtruments, which by curious and ſurprizing operations, give us the moſt exquiſite delight; ſuch as the camera obſcura, the magic lanthorn, the perſpective views of countries, &c. ſo that it is a ſcience of infinite pleaſure as well as profit.*

Were there no other uſe of Optics, but the contrivance of ſpectacles, to help the eyes of old men and purblind people; yet that alone would be ſufficient to recommend the ſtudy of it. Of what infinite advantage are theſe to mankind, not one in a thouſand, but what has need of their help, at ſome time of life, eſpecially in old age. For as the ſight is the moſt noble and moſt extenſive of all our ſenſes; and as we can go about no concerns in life without making uſe of our eyes; certainly the inſtrument muſt be the moſt valuable, which relieves the decayed eyes, and cures their defects, and makes them uſeful, and without which, they would be almoſt uſeleſs. They are certainly in a miſerable condition that have the two windows of the ſoul ſhut up, and are obliged to live in perpetual darkneſs. And ſuch as have them partly obſcured, partake of the ſame miſery in ſome degree. How miſerable would the latter part of men's lives be, if there were no ſuch thing as ſpectacles; and how tedious muſt the lives of old people have been before their invention. Without them a mechanic grown old could not follow his buſineſs, ſuch as watchmakers, engravers, &c. nor a ſcholar converſe with his books, or with his friends by letter. So that this curious invention cannot be too highly valued.

Our natural ſight is confined to very narrow views, and tho' it is very extenſive, yet ſome objects at a diſtance are ſcarce perceptible, while others more remote, cannot be ſeen at all. Here then we have the advantage of teleſcopes, by which we can behold very remote objects very clearly; thus we can have a full view of the heavenly bodies, and by that means underſtand the wonderful mechaniſm of the world; without this inſtrument, all theſe charming ſcenes of nature, ſo far diſtant, would be hid from us. This noble inſtrument then can never be ſufficiently recommended, that has opened a way for our diſcoveries in the heavens, which could never be known by any other means. By this, the whole ſyſtem of nature is laid open to our view. By this we find the ſun to be a great globe of fire; that the planets revolve about him; and that from their different phaſes, they have no light but what they receive from him. By the dark ſpots upon their ſurfaces, are found the times of rotation round their axes. The ſame inſtrument has given us intelligence of the ſatellites of Jupiter and Saturn, which move round theſe planets, in their fixt periodic times; a thing utterly unknown to the ancients. And theſe are of great uſe for determining the longitude; and from thence the motion of light has been proved. Saturn's ring is another diſcovery owing to the teleſcope. The motion of all the heavenly bodies, their diſtances and magnitudes have been hereby found out. The immenſe diſtance of the fixt ſtars is proved by the teleſcope; and by this we diſcover almoſt an infinite number of ſtars more than can be ſeen with the naked eye. Likewiſe all that has been diſcovered of the comets is owing to the uſe of the teleſcope. And by it the times of the beginning and ending of eclipſes are exactly determined, with many other things of like nature, equally aſtoniſhing and curious. The advancement of aſtronomy is wholly from the uſe of *of the teleſcope. Teleſcopes may be alſo uſefully employed upon the earth. The merchant may at a great diſtance diſcern his ſhips upon the ſea. The ſeaman may ſee the enemies ſhips at a diſtance; or he may ſee land from the maſt head, long before he comes at it. Generals may ſurvey the army of his enemy, or his own. And any man may take a view of diſtant towns, cities, buildings, trees, and other objects in the country, eſpecially when he is ſeated on a high place. Theſe are noble diſcoveries of the teleſcope; which, if it had been known to the ancients, would have put an end to all their doubts and diſputes about the ſolar ſyſtem.*

If a teleſcope be fixed in a certain poſition, and thro' it we ſee the top of a tower, &c. at noon, we ſhall ſee the lower part of it at night, or in the morning, owing to a greater refraction at morn and night.

The microſcope diſcovers to us another world of wonders. And here our contemplations may be endleſs, all bodies we look at, ſhewing us ſuch curious contexture of parts, and affording ſuch admirable appearances, and ſuch glorious colours, as cannot be well conceived without ſeeing them.

The very ſmall parts of bodies cannot be diſtinguiſhed by the eye, they ſubtend an angle ſo very ſmall; and if they are brought nearer to ſubtend a greater angle, they come without the limits of diſtinct viſion. For the eye cannot contract itſelf further than for an object at 4 inches diſtance to appear diſtinct. If it is brought nearer, it will appear confuſed; and the more confuſed, the nearer it is. This confuſion may be prevented either by looking thro' a hole in a thin plate of lead, whoſe aperture muſt be the leſs, the nearer the object is; or elſe by interpoſing a microſcope glaſs, between the object and the eye, to look thro'.

To mention in particular ſome diſcoveries made by the microſcope. The ſkin of a human body is compoſed *ed of ſeveral ſcales lying over one another; between theſe layers of ſcales the excretory ducts appear, thro' which we perſpire; there are about 500 of theſe ducts for one ſcale, and 250 ſcales are but equal to a grain of ſand. The particles of duſt flying out of the puff ball, are perfect ſpheres of an orange colour, and their diameters not above* $\frac{1}{50}$ *of an hair's breadth. The circulation of the blood is ſeen by the microſcope, in the fins and tales of fiſhes, and the feet of frogs, and the inſertion of the veins and arteries made viſible. Hence alſo may be ſeen the armour of the flea, the ſpout of the tick, the briſtles of the mite, the motion of the muſcles, and the pulſe in a louſe. Many imperceptible animals may be ſeen with this inſtrument, in green weeds, which grow in water, and in ſtanding water; in the hollow of the cabbage and teazle; and ſwarms of them in the water that drains from a horſe dunghill; ſmall animals lodged in the ſcabs of ſuch as have the itch. It would be an infinite labour to mention all the wonders that this little inſtrument ſhews us.*

Thus it appears, that Optics is not only a ſcience of the moſt extenſive uſe, but likewiſe of the greateſt pleaſure and delight. A great many things in philoſophy can neither be underſtood nor proſecuted without the knowledge of Optics. This gives an account of Light, Colours, Opacity, Tranſparency, Brightneſs, Faintneſs, &c. The ſame ſcience alſo accounts for the appearances of mæteors, the Rainbow, Coronas, Parahelion's, Paraſelene's; it gives account of the nature of ſeeing; without it nothing can be done in aſtronomy. Optics alſo brings forth Perſpective, in which every rule of it is founded.

Spectacles were not found out till the beginning of the 13th century. For the ancients had no knowledge of optic glaſſes, as is evident from their univerſal ſilence in this matter. Not one of them giving the leaſt

leaſt hint thereof. And without doubt an invention of ſuch univerſal uſe to the world could never be concealed. But the uſe of ſingle glaſſes was known 300 *years before men underſtood how to combine them in teleſcopes. They knew nothing of the advantage the eye received thereby, by increaſing its power ſome thouſands of times beyond its natural abilities. For teleſcopes were not invented till about the year* 1600, *nor microſcopes till* 10 *or* 12 *years after.*

Many effects produced by optic glaſſes ſeem wonderful and ſurprizing. A multitude of objects ſeem diffuſed thro' a large ſpace where there is no ſpace; and objects ſeem hanging in the air, where upon trial we can feel nothing. Objects appear ſometimes greater, and ſometimes leſs than they are; and ſometimes pretty, and ſometimes deformed. And they that underſtand the principles of Optics *and* Perſpective, *eaſily underſtand the reaſon of theſe phænomena.* Optics *alſo tends to free the mind from ſeveral prejudices it had imbibed; and when glaſſes are at hand to demonſtrate their effects by experiment, it gives a vaſt pleaſure to the mind, to ſee the* Theory *and* Practice *agree, and at once confirms us in the truth of the* Principles. *And theſe experiments inſinuate to us, the great affinity there is between reaſon and experience; a matter of the greateſt conſequence in all kinds of ſcience; as ſuch an agreement conduces more than any thing elſe, to the certainty thereof, by freeing the mind from all doubt, and fixing a firm aſſent therein.*

From what has been ſaid, and many more things that might be ſaid, appears plainly the excellency of the ſcience of Optics, *and its great uſe to mankind above all others; and how neceſſary it is for us to be acquainted with it, to let us into the ſecrets of nature, not only in regard to the grand fabric of the whole univerſe, but likewiſe to the moſt minute and imperceptible parts of it.*

As to this Treatiſe, I have handled the three different parts of the ſcience diſtinctly; and treated of each of them as clearly as I could; and in as ſhort and conciſe a manner, as I could think on; and have demonſtrated all the principles in a ſhort way, and in a method very natural and plain; omitting nothing that I could think conducive to the advancement of this ſcience, or could find in the moſt approved authors. And in the laſt book have deſcribed ſeveral optical inſtruments, and demonſtrated their powers and effects, which I hope will be acceptable to the reader.

W. Emerſon.

A SYNOPSIS, or ſhort View of the Contents of this Treatiſe.

BOOK I. SIMPLE OPTICS.

OF direct viſion. Of the nature of light, and how the colours are generated by refraction thro' a priſm. The apparent places, magnitude, and brightneſs of objects ſeen directly. The colours of natural bodies how cauſed. Fits of eaſy tranſmiſſion and reflexion of the rays of light. The action of light and bodies upon one another. The motion of light is performed in time. The proportion of moon-light, &c. to day-light.

BOOK II. CATOPTRICS.

The reflexion of light from plane and ſpherical ſurfaces. A general theorem for finding the focus. Of images, their places, diſtances, poſitions, magnitudes, motions, and brightneſs. Burning concaves. The aberration of the rays.

BOOK III. DIOPTRICS.

The refraction of light thro' one or more refracting ſurfaces. The power of different bodies to refract light. Of finding the focus of rays refracted by plane and ſpherical ſurfaces. A general theorem for finding the focus of rays refracted by a ſpherical ſurface. General theorems for finding the foci, for all ſorts of lenſes. The conjugate foci of all lenſes determined. To find the focus, and the vertex of a lens by experiment. Images made by refraction their places, diſtances, poſitions, magnitudes, ſhapes, motions, and brightneſs. Burning glaſſes. Aberration of the rays.

BOOK IV. OPTICAL INSTRUMENTS.

The combination of glaſſes. Of viſion, and the ſtructure of the eye. Teleſcopes with 2, 3, 4, glaſſes; their magnifying powers, diſtinctneſs, brightneſs, and viſual angle. Table of their dimenſions. The ſolar teleſcope. Teleſcopic ſights, and taking angles. Teleſcopes without tubes. Reflecting teleſcopes; Gregory's, Caſſegrain's, Newton's; their magnifying powers, diſtinctneſs, brightneſs, and viſual angle. Tables of their dimenſions. To find the magnifying power of a teleſcope by experiment. A ſingle microſcope, its magnifying power and brightneſs. A double microſcope, its magnifying power, brightneſs, and viſual angle. A reflecting microſcope. The ſolar microſcope. The magic lanthorn. Portable camera obſcura, ſeveral kinds of them. Several machines for ſhewing perſpective views. Explanation of the colours of the rainbow.

OPTICS.

DEFINITIONS.

DEF. I.

OPTICS, is a ſcience which teaches the nature of viſion; and of every thing relating to the ſight.

DEF. II.

A *Ray* of light, is an extremely ſmall part of it moving in a right line, or otherwiſe.

DEF. III.

A *Beam* of light is a ſenſible part of it, conſiſting of a great number of rays.

DEF. IV.

A *Pencil* of rays, is a portion of light of a conical form, proceeding from a point, or elſe tending to a point.

DEF. V.

Parallel Rays, are ſuch as run in parallel directions; and *converging rays*, are thoſe that tend to a point; and *diverging rays*, are ſuch as proceed from a point.

DEF. VI.

A *Medium*, is whatever the rays of light paſs thro', as water, oil, glaſs, air, or any tranſparent body; and even a vacuum.

DEF. VII.

Reflexion, is when a ray of light flies off from the ſurface is falls upon, without entering into it.

DEF. VIII.

Refraction, is the bending or breaking of a ray of light, where it goes out of one medium into another.

DEF. IX.

The *Angle of Incidence*, is the angle made by the incident ray, and the perpendicular to that point of the ſurface on which the ray falls.

DEF. X.

The *Angle of Reflexion*, is the angle comprehended between the reflected ray, and the perpendicular on that point of the ſurface, on which it falls.

DEF. XI.

The *Angle of Refraction*, is the angle which the refracted ray contains with the perpendicular to the refracting ſurface, at the point of incidence. And the *refracted angle*, is that contained between the incident and refracted rays.

DEF. XII.

The *Focus*, is a point in which all the reflected or refracted rays meet together. The point from which the rays ſeem to diverge, is called the *virtual Focus*. The point to which parallel rays are reflected or refracted, is the *principal Focus*.

DEF. XIII.

The *focal Diſtance*, is the diſtance of the focus from the reflecting or refracting ſurface. This generally relates to the principal focus.

DEF. XIV.

Conjugate Foci, are two ſuch points, that either of them being the radiant point, the other will be the focus.

BOOK I.

Of DIRECT VISION, *and ſuch Properties of Light, as are deduced from Experiments.*

PROP. I.

WHEN the eye views many objects at great diſtances, it refers all their places to the ſurface of a ſphere, whoſe center is the eye itſelf.

For as far as an eye can ſee diſtinctly in any one direction, ſo far it can ſee in any other direction, if not hindered by intermediate bodies; and this it can do quite round. Hence, when the objects we look at, are at great diſtances, it is natural to ſuppoſe them at equal diſtances. For under equal circumſtances, nothing appears to make one diſtance to be judged greater than another. Therefore we take the ſimpleſt way, and ſuppoſe them all equidiſtant, or conſider them as ſo. Whence theſe diſtances being all equal or ſuppoſed equal, and all proceeding from one point, it it plain they are the radii of a ſpherical ſurface, whoſe center is that point. That is, the eye places all theſe objects in the ſurface of a ſphere whoſe center is the eye itſelf.

Cor. 1. *The eye is always placed in the center of its own view.*

Cor. 2. *Any portion of the heavens which falls under our view, appears as the portion of the concave ſurface of a ſphere.*

Fig. Cor. 3. *When we look at the ſun, moon, and ſtars; or even at the clouds; they appear to us to be ſituated in the ſurface of one and the ſame ſphere.*

For theſe objects being beyond the extent of diſtinct viſion, therefore the eye cannot judge of their true diſtances. Therefore they will appear to us, as all ſituated in the ſurface of the ſame ſphere, and ſo equally diſtant. Tho' they be really at different diſtances from us.

Cor. 4. *Hence if the eye was to have a full view, from any place on the ſurface of the earth; it could ſee no more than a hemiſphere.*

For the horizon cuts off the other hemiſphere; and the body of the earth below that, hinders us from ſeeing any thing at a great diſtance.

Cor. 5. *The eye, keeping a fixed poſition, can ſee no more objects than are contained in a hemiſphere lying directly before the eye, nor quite ſo much.*

For if the baſe of this hemiſphere is ſuppoſed to touch the pupil of the eye; no rays can enter below that baſe; nor even in the plane of this baſe or near it, to cauſe any diſtinct viſion.

PROP. II.

Such rays as enter the eye perpendicularly, have the greateſt power to cauſe diſtinct viſion.

This is matter of obſervation. And it is plain that rays that are oblique have leſs force to produce their effects; and may be ſo oblique as to produce no ſenſible effect at all. Therefore thoſe rays that are moſt direct, are moſt efficacious in cauſing viſion; as they will fall perpendicularly upon the retina, at the bottom of the eye. And by theſe the image of any object will be painted moſt diſtinctly upon the retina.

Cor. *Hence, we ſee objects the moſt diſtinctly, which are directly before our eyes.* Fig.

This is plain from this Prop. And is confirmed by experience, as is evident by our turning our eyes directly to the object; or from one part of an object to another, in order to ſee it clearer.

PROP. III.

An object appears greater or leſs, according as it is ſeen under a greater or leſſer angle. 1.

Let AC*a* be the axis of the eye; AB, DE two equal objects. To the eye at C, draw BC*b*, and EC*c*. Then the object BA appears under the angle BCA, or *bca*, and ED under the angle ECD or *e*C*a*. But *e*C*a* is greater than *b*C*a*; and *ae* greater than *ab*. But *ae*, *ab* are the appearances or pictures of the objects DE, AB, upon the retina. Therefore the former appears greater than the latter.

Cor. 1. *The diſtance between two objects* A *and* B, *appears to be greater as the diſtance of the ſpectator is leſs.*

For at the diſtance CD, the diſtance of the objects appears under the angle ECD or *e*C*a*. And at the diſtance AC, it appears under the angle BCA or *b*C*a*.

Cor. 2. *The apparent linear magnitude of an object* AB, *is reciprocally as the diſtance* CA.

For AB appears no greater than DF. But DF : DE or AB : : CD : CA. That is, the apparent magnitude of AB : apparent magnitude of DE : : CD : CA.

Cor. 3. *The ſuperficial apparent magnitude of an object, is reciprocally as the ſquare of the diſtance.*

Fig. 1. For the apparent hights will be reciprocally as the diſtance. And for the ſame reaſon, the apparent breadths will alſo be reciprocally as the diſtance. Therefore the apparent ſurface compounded of theſe, will be reciprocally as the ſquare of the diſtance.

Cor. 4. *A perſon, eſpecially at great diſtances, judges that object to be greateſt; which appears under the greateſt angle.*

For nature always takes the eaſieſt way; and it's eaſier to ſuppoſe things at the ſame diſtance, than at different diſtances, when nothing appears to the contrary. And at equal diſtances, a greater object will appear under a greater angle.

PROP. IV.

An object appears ſituated in that place, from which the rays laſt of all diverge, in coming to the ſpectator's eye.

For by frequent experience, a man always finds an object ſituated in that place from which the rays diverge, and come directly to his eye. And being uſed to this, he judges ſo always. For if the rays had been brought there by reflexion or refraction, they make the ſame picture in the bottom of the eye as if they had come directly from that place. And all viſion being performed by the pictures upon the retina; when theſe are alike, we muſt judge the ſituation of the object to be alike. Thus an object which appears by reflexion in a looking glaſs appears behind the glaſs; and an object ſeen thro' a ſpectacle glaſs appears further off. And a ſhilling in the bottom of a veſſel of water appears higher, and the water appears ſhallower, than it is.

PROP.

PROP. V.

From every point of a luminous object, the rays of light diverge, and flow every where in right lines, thro' the ſame medium.

For if the rays do not flow from every part of the object, ſuch parts as have none flowing from them cannot be viſible, ſince nothing can be ſeen without light, either innate or reflected. And it flows in all manner of directions, becauſe in any ſituation of the eye the ſame part of the object appears and may be ſeen. And being viſible in all ſituations, the light muſt flow in all directions. And it muſt flow in right lines in the ſame medium, for no reaſon can be given why it ſhould deviate more to one ſide than another. Experiments likewiſe confirm that it moves in right lines.

Cor. 1. *No rays of light are naturally converging; but always diverge.*

For the rays are projected from bodies, or the parts of bodies; whether the light is innate or not; and therefore naturally diverge. When they converge, they are made to do ſo by art, by means of glaſſes, as mirrors, lenſes, &c.

Cor. 2. *Rays that ſhew things far diſtant, fall upon the eye parallel. And to ſhew things near hand, they diverge from a point; which to cauſe diſtinct viſion, ought to have a proper diſtance, as* 6, 8, 12 *inches, &c. or as near as the eye will permit.*

Cor. 3. *And therefore to make an object ſeem at a great diſtance; the rays flowing from each point of it ought to be brought parallel to the eye.*

This is to be done by the help of proper glaſſes; and then art will imitate nature.

Fig.

PROP. VI.

2. *The denſity of rays flowing from an object, or any part of an object; is reciprocally as the ſquare of the diſtance.*

Let the rays flow from A, and fall upon the plane BC; then taking the plane BC away, let the ſame light fall on any other plane DE, which is done by producing the lines AB, AC, to D and E. Then the area of the ſurface BC, is to the area of the ſurface DE; as BC^2 to DE^2; or as AB^2 to AD^2. Let s be any ſmall ſpace, l the quantity of light contained in DAE. Then $BC^2 : s :: l : \frac{ls}{BC^2} =$ the number of rays on the ſpace s, at the diſtance AB. And $DE^2 : s :: l : \frac{ls}{DE^2} =$ number of rays on the ſpace s, at the diſtance AD. And the denſity of light being as the number of rays; it will be, denſity of light at B : denſity of light at D $:: \frac{ls}{BC^2} : \frac{ls}{DE^2} :: DE^2 : BC^2 :: AD^2 : AB^2$.

Cor. 1. *The quantity or intenſity of light, flowing from a luminous object, is reciprocally as the ſquare of the diſtance.*

Cor. 2. *The brightneſs of an object, is the ſame at all diſtances from the eye; if none of the rays be ſtopt by the way.*

For the brightneſs is the ſame, when the ſame quantity of light falls upon the ſame quantity of the retina, in the eye; or when the denſity of light on the retina is the ſame. But the denſity of light falling on the pupil, or the quantity of light falling on it, is reciprocally as the ſquare of the diſtance. And (III. Cor. 3.) the apparent magnitude of the object is reciprocally as the ſquare of the diſtance. Therefore the quantity of light on the pupil, is as the apparent magnitude of the object, that is, as the ſpace taken up on the retina. Therefore the quantity of light being as the ſpace taken up, its denſity is given; and therefore the brightneſs is given, or remains the ſame. Fig. 2.

Cor. 3. *Hence the faintneſs of remote objects is owing to the denſity and opacity of the atmoſphere, which ſtops a great part of the light, in coming to the eye.*

PROP. VII.

If a room be made quite dark, and a ſmall hole in the window ſhut be made. The objects out of the houſe being ſtrongly illuminated; the rays proceeding from them, thro' the hole, will form their pictures on a paper, or on the oppoſite wall, in their proper colours; but in an inverted poſition. 3.

Let PQR be the dark room, F the hole in the window ſhut, ABCD an object without. The rays proceeding from the top A, in the right line AF, will form its image at *a* towards the bottom of the wall. And the rays proceeding from the bottom C, in the line CF, will paint its image at the top *c*. Likewiſe the rays proceeding from the left B, in the line BF, will make the image of B on the right hand at *b*. And thoſe proceeding from the right hand D, in the line DF, will form its image on the left at *d*. Likewiſe all the rays proceeding from the line AC, will form its image *ac*. And thoſe from BD will make its image *bd*. And the like for any other object expoſed to the hole F.

Cor.

Fig. 3. Cor. 1. *If the hole* F *be very ſmall, the pictures will be very diſtinct, but hardly viſible for want of light. And if the hole be large, the pictures will be brighter, but not diſtinct. To remedy theſe things, a broad convex lens of a proper focus, muſt be placed in the hole made wide enough to receive it.*

Cor. 2. *If an object, too near the eye, appear confuſed; it may be ſeen clearly thro' a hole made in a piece of paper with a pin, and put cloſe to the eye. Then it will appear diſtinct, and in its true poſition.*

This is upon the ſame principle. For if the hole is ſmall, the rays flowing from all the points of the object, will fall upon as many diſtinct points of the retina; and make a diſtinct picture.

Cor. 3. *But if the paper with a ſmall hole in it, be held at a ſmall diſtance, and a needle held cloſe to the eye, or any way between the eye and the paper; it will appear inverted, and magnified.*

The reaſon of this is, that the rays croſſing one another at the hole, before they come to the object; the picture of the object muſt be painted the ſame way that it lies; contrary to the caſe of the 2d Corol. where the rays croſſing one another between the object and the eye, makes the picture inverted. But we know, that a picture inverted in the bottom of the eye, makes the object ſeem erect; conſequently, when the picture is erect in the bottom of the eye, the object will ſeem inverted.

42. Cor. 4. *From the ſame principle it follows, that if the ſun ſhines thro' a long narrow ſlit* AB *in a window ſhut; its image* CD *at a good diſtance from the window, will approach to a circle.*

For the cone of rays flowing from all parts of the ſun thro' the point A, will be projected into a circle at C; and thoſe flowing thro' B will be projected into a circle at D. And ſo rays flowing thro' all points between A and B, are projected into innumerable circles between C and D; each of theſe circles is the image of the ſun, and all together make up the oblong figure CD. In this figure the extreme circles C, D, grow larger at a greater diſtance, whilſt the diſtance of their centers remains the ſame, being equal to AB. So that as the diſtance increaſes, the length and breadth of the figure increaſes equally, and at a great diſtance, their difference vaniſhes as to ſenſe. At the place FG, the figure is narrow, having rectilinear ſides and circular ends. And after the ſame manner, if the ſun ſhines thro' a triangular hole or any other figure, the image at a great diſtance will be nearly circular, and the centers of the extreme circles will form a triangle or other figure, ſimilar and equal to the hole. Fig. 42.

SCHOLIUM.

An apparatus made after the ſame manner deſcribed in this Prop. is called a *Camera obſcura.*

PROP. VIII.

The light of the ſun conſiſts of rays differently refrangible.

Ex. 1.

To prove this by experiment. In a very dark chamber, make a round hole F, about $\frac{1}{4}$ inch diameter, in the window ſhut EG. Place a glaſs priſm ABC very near the hole; ſo that a beam of light SF coming in at the hole, may paſs thro' the priſm and be refracted upwards, towards the oppoſite wall of the chamber, and there form a coloured image of the ſun PT. This priſm is triangular at each end, 3 or 4 inch long, and is poliſhed on the 3 ſides AB, BC, CA. It is here placed parallel to the horizon, and its axis perpendicular 4.

Fig. 4. dicular to the beam SF. Turn the priſm ſlowly about the axis, and you will ſee the refracted light on the wall, or the coloured image of the ſun, firſt to deſcend, and then to aſcend. Between the deſcent and aſcent, when the image ſeems ſtationary, ſtop the priſm and fix it there. For in that poſture the refractions of the light on the two ſides CB, CA, are equal to one another.

So likewiſe in other experiments, when you would have the refractions on both ſides the priſm to be equal, you muſt note the place on the wall where the image ſtands ſtill, between two contrary motions; and make it faſt there. In this poſture of the priſm all the following experiments are to be made, unleſs ſome other poſture is deſcribed. The priſm remaining in this poſture, let the refracted light fall perpendicularly upon a ſheet of white paper at the oppoſite wall of the chamber, and you'll ſee the ſolar image formed on the paper to be oblong, and not oval, but terminated with two rectilineal and parallel ſides, and two ſemicircular ends; the ſides bounded pretty diſtinctly, but the ends very confuſedly and indiſtinctly, the light there decaying, and vaniſhing by degrees. Here the breadth anſwers to the ſun's diameter; at 18 $\frac{1}{2}$ feet from the priſm, the breadth diminiſhed by $\frac{1}{4}$ inch, the diameter of the hole, is 1 $\frac{7}{8}$ inch, and ſubtends an angle at the priſm of half a degree, which is the ſun's apparent diameter. The length 10 $\frac{1}{4}$ inches, when the refracting angle of the priſm C is 64 degrees. With a leſs angle of the priſm the length of the image will be leſs. If the priſm be turned about its axis, that way which makes the rays emerge more obliquely to the ſide AC; the image ſoon becomes an inch or two longer, or more. And if the priſm be turned about the contrary way, ſo as to make the rays fall more obliquely on the ſide CB, the image ſoon becomes

comes an inch or two ſhorter. And therefore in trying this experiment, one muſt be as careful as can be, in placing the priſm, ſo that the refractions on both ſides may be alike. Fig. 4.

This experiment tried with a priſm, whoſe refracting angle C is 62 $\frac{1}{2}$ degrees, the length of the image will be 9 $\frac{3}{4}$ or 10 inches, at the ſame diſtance. The magnitude of the hole in the window ſhut, the different thickneſs of the priſm, where the rays paſs thro', the different inclinations of the priſm to the horizon (or different hights of the ſun), make no ſenſible changes in the length of the image. Neither does the different matter of the priſm make any; for in a priſm made of plates of glaſs and water within, there is the ſame reſult.

Alſo the rays always go on in right lines from the priſm to the image. And therefore at their going out of the priſm, have all that inclination to one another, which cauſes the length of the image; that is the inclination of 2 $\frac{1}{2}$ degrees or more. From all which it is plain, the rays falling on P are more refrangible than thoſe falling on T. For if the rays were all alike refrangible, the image PT would be a circle, whoſe length is equal to its breadth; which is eaſily proved thus.

Let SKHP be a ray coming from the lower part of the ſun to the higher part of the image; and refracted by the priſm at K and H, SLIT a ray which comes from the higher part of the ſun, to the lower part of the image, and is refracted at L and I. Suppoſe the refraction on both ſides the priſm, are equal; we have, refraction at K = refraction at I, and refraction at H = refraction at L. Then by addition, the refractions at K and H = refractions at I and L. And therefore the two rays being equally refracted, have the ſame inclination to one another after refraction as before; that is, half a degree, equal to the ſun's apparent diameter.

Fig. 4. diameter. So then PT would ſubtend an angle at the priſm of half a degree, and therefore would be equal to the breadth, and conſequently the image would be round. But inſtead of that, it is five times longer than it is broad.

Likewiſe the image PT is coloured; for from T to P it has theſe different colours in this order, red, orange, yellow, green, blue, purple, violet; at the places, *r*, *o*, *y*, *g*, *b*, *p*, *v*, reſpectively. So that the moſt refracted part at P is violet, and the leaſt refracted at T is red. And the intermediate part, all the other colours.

Exp. 2. When a beam of light is let in at the hole F, of the window ſhut; hold the priſm in ſuch a poſition that its axis may be perp. to the beam. Then look thro' the priſm to the hole, and turning the priſm about back and forward, to find when the image is ſtationary; there ſtop the priſm, the refraction on both ſides being then equal. Then looking thro' it, at the hole; the length of the image will appear to be many times greater than the breadth. The moſt refracted part being violet; and the leaſt refracted, red. The middle parts blue, green, yellow in order.

And if the priſm be removed out of the ſunbeam, and then looking thro' it at the hole; you will have the ſame appearance. Now if all the rays were refracted alike, the hole would appear round when refracted thro' the priſm. So that at equal incidencies there is a conſiderable inequality of refraction.

5. *Exp.* 3. To try what is the effect of refracting the rays a ſecond time. Things being ordered as in the firſt experiment, place a ſecond priſm cloſe by the firſt in a croſs poſition as DH, that it may again refract the beam which comes thro' the firſt priſm. The firſt priſm refracts it upwards; and the ſecond, ſideways. By the firſt priſm alone the

beam is refracted to PT, and *qt* is the image made Fig. by the refraction of the two croſs priſms together. 5. Then it will appear by the refraction of the ſecond priſm that the breadth of the image is not increaſed, but the upper part P ſuffered a greater refraction in both priſms, and the lower part T, a leſs refraction in both priſms; and that without any dilatation of the image in breadth. Likewiſe as before, the upper part *q* appears violet; and the lower part *t*, red; and the reſt as before. So that here is no dilatation or ſplitting of rays.

And if a third and even a fourth priſm be placed in the ſame manner after the ſecond, the reſult will be the ſame; and the moſt refrangible rays will be moſt refracted, as before; and the leaſt, the leaſt refracted, whilſt their colours remain unchanged.

Exp. 4. If inſtead of one hole in the window, 6. there be made two holes, under one another, at a ſmall diſtance; and refracting the light upwards by two parallel priſms, placed one at each hole; and then refract the two images to the oppoſite wall, in ſuch manner that the two coloured images may be joined end to end in a right line. Then if both theſe refracted beams were again refracted ſideways by a third priſm placed acroſs the two firſt (as in Exp. 3.). Then the two beams would be refracted to the places *pt*, *mn*; which would not now lie in one right line with their ends touching as before; but be ſeparated and become parallel. The blue ends *m*, *p* being by a greater refraction tranſlated further from its former place, than the red ends *n*, *t*. And the ſame happens if the third priſm be placed at a good diſtance from the others.

Exp. 5. If a beam of light be reflected by one ſide of a priſm, and then refracted by another priſm, ſome of the rays will be more refracted, and others leſs. Or if it paſſes thro' one ſide of a priſm, Fig. a priſm, and is reflected by another ſide, and made 6. to paſs thro' the third ſide; and after this be refracted by another priſm; ſome rays will at laſt be more refracted than others. Or if a beam paſs thro' parallel ſurfaces of glaſs, and be then refracted; the conſequence will ſtill be the ſame. And therefore it is fully proved, that the ſun's light conſiſts of rays differently refrangible.

4. *Exp.* 6. Let a ſmall hole be made in the paper where the image PT is; and let each of the colours paſs ſucceſſively thro' this hole; and be refracted again by a priſm behind the hole. Then each of theſe colours will be found to continue the ſame as before without any manner of alteration. And the figure or ſpot of light on the wall, will, as to ſenſe, be circular. So that none of theſe colours are the leaſt changed by refraction. Likewiſe if ſmall bodies be placed in this new light, they will appear of the ſame colour as that light, red in the red light, green in the green light, &c. So that the colours are no way changed by reflexion. Likewiſe if you look at any of theſe ſpots of new light upon the wall, thro' a priſm, they will appear wholly of the ſame colour, and no way dilated or expanded in length. So that homogeneal light ſuffers no manner of alteration in any caſe. And in all theſe trials it will appear that thoſe rays, which were moſt refracted at firſt, are always afterwards moſt refracted. And thoſe that are leaſt at firſt, are always leaſt afterwards.

Cor. 1. *Every ray of light has a peculiar degree of refrangibility, which cannot be changed by any reflexions or refractions; but remains conſtantly and invariably the ſame.*

Cor. 2. *The confuſed viſion of any object ſeen thro' a refracting body or medium, ariſes from the different refraction of the light.*

For different rays flowing from the ſame point Fig. of an object, will be refracted to different points on the retina at the bottom of the eye.

Cor. 3. *The light reflected or emitted from all bodies, conſiſts of rays differently refrangible.*

For the light reflected from all bodies on the earth, is the ſun's light. And beſides, if one looks at any object thro' a priſm, that object will appear tinged with colours. Take a ſmall part of a body, illuminate it ſtrongly, and looking at it thro' a priſm, you'll have an oblong image like PT (*fig.* 4.) with all the colours. In like manner, a ſtar, a lamp, a candle, a burning coal, red hot iron, or any burning matter ſeen thro' a priſm, gives the ſame appearance.

Cor. 4. *Hence alſo the ſimple primary colours are ſeven; red, orange, yellow, green, blue, purple, violet. The red being leaſt refracted, and ſo in order to the violet, which is moſt. And the ſpaces taken up by theſe, are not equal; the orange and purple being little more than half each of the reſt, which are nearly equal.* 4.

This is known by meaſuring the parts of the image PT.

Cor. 5. *Hence objects ſeen thro' glaſſes in purified or homogeneal light will appear diſtinct; but very confuſed in heterogeneal or common light.*

This is plain from the 3d experiment; and may be further evinced by placing flies, or the letters of a book, or any minute objects in homogeneal light, where they will be perfectly ſeen, and their parts diſtinctly defined; but will appear confuſed in common light, and their ſmall parts cannot be diſtinguiſhed therein.

SCHOLIUM.

Sir J. Newton, by accurately meaſuring the ſpaces, that the ſeveral colours take up, found them as follows;

red,	orange,	yellow,	green,	blue,	indigo,	violet.
$\frac{1}{8}$,	$\frac{3}{40}$,	$\frac{2}{15}$,	$\frac{1}{6}$,	$\frac{1}{6}$,	$\frac{1}{9}$,	$\frac{2}{9}$.

Where the whole length of the image is 1.

PROP. IX.

Rays of light which differ in colour, differ alſo in degrees of refrangibility.

7. *Exper.* 1. Take an oblong piece of paper DGI, draw a line croſs it in the middle, to divide it into two equal parts. Paint one end DG with blue, the other GI with red, as ſtrong as you can. Lay it flat where there is light enough; look at it thro' a glaſs priſm, held parallel to it, and to the horizon; then if the refracting angle of the priſm be upwards, ſo that the paper may ſeem to be lifted upwards by refraction; the blue half FO will be lifted higher by refraction, than the red half OK. But if the refracting angle of the priſm be turned downwards, ſo that the paper may ſeem to be carried lower by refraction; its blue half will be carried lower than the red half. Therefore the light from the blue is more refracted, and therefore is more refrangible, than the light from the red.

Exp. 2. Take the ſame paper, whoſe two halves are painted with red and blue, and lap a thread of black ſilk ſeveral times about it, ſo that the parts of it may appear upon the colours like ſo many black lines drawn over them. Set this paper perpendicular againſt a wall in a dark room, or in the night; ſo that one of the colours may ſtand to the right, and the other to the left. Place a candle near the paper to illuminate it ſtrongly. Then take a lens of a large focal diſtance, and place it at ſuch a diſtance, that the rays coming from the ſeveral points of the paper may be collected to a focus, at about the ſame diſtance on the other hand; and ſo form the image of the coloured paper upon a white paper placed there. Then moving the paper to and from the lens, to find where the images of the blue and red parts of the paper appear moſt diſtinct; which is eaſily known by the images of the black threads of ſilk, which will be diſtinct there; and you'll find where the red half of the paper appears diſtinct, the blue half will appear confuſed; ſo that the black lines on it can ſcarce be ſeen. And on the contrary, where the blue half appears diſtinct, the red half will be confuſed, ſo that its black lines can ſcarce be ſeen. And therefore between theſe two places, where theſe ſeveral images appear diſtinct, there will be a remarkable diſtance, which will be about an inch and a half, if the diſtance of the candle or paper from the lens be about ſix feet. Therefore the focal diſtance of the red being greater, than that of the blue; proves the blue rays to be more refrangible than the red. Therefore rays that differ in colour, differ alſo in their degrees of refrangibility. The ſame appears by the experiments in the laſt Prop. Fig. 7.

Note, A lens is a ſolid piece of glaſs, the two ſides being the ſegments of ſpheres, and poliſhed.

Cor. 1. *Hence all light naturally conſiſts of heterogeneous rays, which exhibit different colours.*

Cor. 2. *Every ſort of homogeneal rays produces its proper colour, according to its degree of refrangibility. And thoſe that agree in colour, agree in the degree of refrangibility; and its colour cannot be changed by any reflexions or refractions.*

Fig. 7. Cor. 3. *There are as many ſimple colours as there are degrees of refrangibility, and therefore an infinite number.*

For tho' in Exp. 1. Prop. VIII. there are only ſeven ſimple colours mentioned; yet each of them conſiſts of an infinite number of degrees, and therefore are infinitely various.

Cor. 4. *The perfection of teleſcopes and microſcopes is prevented by the different refrangibility of the rays of light.*

This appears by the 2d experiment; for theſe inſtruments are compoſed of lenſes of ſeveral ſorts. No rays iſſuing from a point can be refracted exactly to a ſingle point.

Cor. 5. *Hence it is plain, if the ſun's light conſiſted but of one ſort of rays; there would be but one colour in the whole world, or all things would be of one colour. Nor would it be poſſible to produce any new colour by reflexion or refraction. And of conſequence the variety of colours depends upon the compoſition of light.*

SCHOLIUM.

4. If a large hole F be made in the window ſhut EG, and the priſm ABC placed near it, and cloſe to the priſm on the other ſide, a thin board DO, with a ſmall hole in it; and at 12 feet diſtance, the board MN be placed, having a ſmall hole in the middle of it; and cloſe behind it, the priſm RQ. Then if the priſm ABC be turned ſlowly to and fro about its axis, the image PT will move up and down on the board MN; and by this means, all the ſeveral parts of it may be made to paſs thro' the hole in it. And each of theſe paſſing thro', is refracted again by the priſm RQ, and ſo paſſes to the oppoſite wall. Then it will appear, that the light which is moſt refracted at ABC, is alſo moſt

Fig. 4. refracted at RQ, and goes to the higher part of the wall at X. And the light which is leaſt refracted at ABC, is alſo the leaſt refracted at RQ, and goes to the lower part of the wall at Y. And that the moſt refracted at X is violet, and the leaſt refracted at Y is red. And all this happens, whilſt the two boards and the priſm RQ remain unmoved; by which means the incidence of the rays upon the ſecond priſm, remains always the ſame. Therefore at the ſame incidence, ſome rays are more refracted than others; and thoſe that are more refracted at firſt, are more refracted afterwards, and have always the ſame colour annext; and the like for the reſt. So that it is evident, that thoſe rays that are endued with different colours, have different degrees of refraction. From the perſpicuity and evidence of this experiment, it is called the *Experimentum crucis.*

PROP. X.

All rays of light falling on a reflecting ſurface, in the ſame angle; are reflected in equal angles, if they be reflected at all.

8. *Exper.* Let F be a hole in the window ſhut, G a reflecting ſurface placed horizontally. Let a beam of light from the ſun FG, come thro' the hole F, and fall upon the reflecting ſurface G, and then be reflected to H on the oppoſite wall; the figure of the reflected light at H will be circular like the hole F. And therefore there is no ſeparation of the rays as in refraction. And no colours are brought by reflexion.

Alſo if a paper be held any way between G and H parallel to the wall at H; the image received upon the paper will always be ſimilar to the image at H, and of a circular form. And therefore the rays that had the ſame incidence, will run parallel

Fig. 8. to one another after reflexion; and conſequently are reflected at equal angles.

Indeed the Beam FG grows wider as it is further from F, and GH grows bigger as it is further from G; but this is only owing to the apparent magnitude of the ſun's diameter. For from any one point of the ſun's diſc, the rays which fall parallel on G, will be reflected parallel to H.

9. Cor. *Hence there is no ſeparation of the rays of light by reflexion, any thing like what there is by refraction.*

Indeed the moſt refrangible rays will be the ſooneſt reflected, if they move out of a denſe medium into a rare one; as ſuppoſe a beam of heterogeneous light moves in direction FG; ſome of it is refracted along GD, and ſome along GC. Therefore if the angle FGA be diminiſhed; the way of the moſt refrangible rays, GC, will come to GB, and be reflected along GH. But this is nothing to the preſent Prop. For it is reflected, becauſe it is impoſſible it can be refracted any longer, as it cannot enter the medium; and in this ſenſe, the moſt refrangible rays are the moſt reflexible.

PROP. XI.

Whiteneſs, or the ſun's light, is compounded of all the primary colours mixed in a due proportion.

4. *Exp.* 1. A beam of light being refracted by a priſm, as in Ex. 1. Prop. VIII, and made to form the image P on a piece of paper. Take away the paper, and hold a lens in the refracted rays, ſo as they may all fall upon it. This will cauſe the coloured light, which diverged from the priſm to converge, and meet again at its focus. If you hold a paper behind the lens, the ſolar image will appear upon it intenſely coloured; and that by re-

ceding from the lens, thoſe colours will come more and more towards one another; and by mixing together, will more and more dilute one another continually; till at length the paper comes to the focus, where by a perfect mixture, all the colours will wholly vaniſh, and be converted into whiteneſs; the whole light appearing now upon the paper, like a little white circle. And afterwards by moving farther from the lens, the rays which converged and met at the focus, will now croſs one another there, and diverge from thence, and thereby make the colours to appear again, but in a contrary order, the red being now above, and the violet below. Here then the ſun beam SF, which at firſt was white, is by the refraction of the priſm ABC reſolved into its original colours; theſe colours, by the refraction of the lens, are again intimately mixt, and reduced to their original whiteneſs. Fig. 4.

Now it is evident this whiteneſs is made up of all the colours of the image. For if any of the colours be ſtopt at the lens, the whiteneſs will ceaſe, and degenerate into that colour which is made by the compoſition of the remaining colours. And if the intercepted colour be then let paſs, and fall upon that compound colour, it will immediately reſtore its whiteneſs by mixing with them. Thus if the blue rays be intercepted, the reſt will appear reddiſh, and then letting the blue paſs, the whiteneſs is reſtored. And if the red be ſtopt, the reſt will appear blueiſh; and letting the red paſs, the whiteneſs is again reſtored.

And in making this white, the ſeveral rays do not ſuffer any change by acting upon one another. For if the paper be held beyond the focus, and the red colour be ſtopt and then let paſs again, the violet colour on the paper ſuffers no change. Nei-

Fig. ther will the red be changed by ſtopping and let-
4. ting paſs the violet.

When the paper is held in the focus, and the white image there be viewed thro' a priſm, you'll have a coloured image like PT. And if any colour at the lens be ſtopt and let paſs by turns, it will diſappear and return as often; but the remaining colours will not ſuffer any change. One colour then depends on one ſort of rays, and another colour, on another ſort.

10. *Exp.* 2. Make an inſtrument XY in faſhion of a comb, whoſe teeth are $1\frac{1}{2}$ inch broad, and diſtance two inches; and interpoſing ſucceſſively the teeth of this inſtrument near the lens MN, placed as in the laſt experiment, you will ſtop part of the colours by the teeth, whilſt the reſt go on, thro' the ſpaces between the teeth, to the paper DE, in the focus, where it makes a round image. This image appears white, when the comb is taken away; but the comb being interpoſed, the whiteneſs changes its colour to that which goes on thro' the comb. And the colour is perpetually ſo varied by the motion of the comb up and down, that in paſſing of all the teeth over the lens, the colours red, yellow, green, blue, purple, do always ſucceed one another, when the motion is ſlow. But if the comb be moved very quick, that the colours by their quick ſucceſſion cannot be diſtinguiſhed from one another, the appearance of the ſingle colours ceaſes, and from a confuſion of them all, there ariſes one uniform white colour. Therefore of the whole light which now appears white, by the mixture of all the colours, there is no part really white. One part is red, another is yellow, a third green, a fourth blue, a fifth purple. If the impreſſions of theſe ſeveral colours follow one another ſlowly, ſo that they may be ſeverally perceived,

there is made a diſtinct ſenſation of all the colours Fig. ſucceſſively. But if the impreſſions follow one ano- 10. ther ſo quickly, that they cannot be ſeverally perceived, there ariſes out of them all one common ſenſation, which is neither of this nor that colour, but is indifferent to them all; and this gives a ſenſation of whiteneſs. Therefore when all the colours are viewed perfectly mixt in the focus of the lens, this will much more ſtir up a ſenſation of whiteneſs.

Exp. 3. A beam of light being refracted by a 4. priſm ABC, and the ſun's coloured image PT made to fall on the oppoſite wall. If a ſpectator ſtand at FG, and place another priſm cloſe by ABC, and in the ſame poſition, and look thro' it to the image PT; then the colours of it will appear to be contracted into a round ſpot perfectly white. Which therefore is made ſo by the mixture of all the colours together.

Exp. 4. Take a large circle or wheel ADF. Di- 11. vide its circumference into 12 equal parts; ſet two of them from A to B; that is, make AB two of them, BC 1, CD 2, DE 2, EF 1, FG 2, GA 2. Draw lines to the center O. Then paint it, making DOE red, EOF orange, FOG yellow, GOA green, AOB blue, BOC purple, and COD violet; all of as bright colours as can be had. Then put an axis thro' the center O, and cauſe the wheel to revolve ſwiftly about its axis; and none of the colours will then appear ſingly; but the whole will appear to be white. And this white will be the fuller, as the particular colours are the brighter.

Exp. 5. Whiteneſs may be made by mixing coloured powders together. Thus, put together orpiment, a purple powder, biſe, and verdigreaſe, theſe being mixt well together, will compound a white colour; the quantities of them muſt be found out by trials, and leſs or more of each muſt be taken, Fig. taken, according to their brightneſs or dullneſs. But all powders of this kind, loſe or ſtifle a great deal of the light that falls upon them; and therefore a white cannot be made bright enough of theſe, but will appear duſky; to help this, it muſt be placed in a ſtrong light, and viewed at a diſtance. From all theſe experiments it appears, that whiteneſs is a due mixture of all other colours.

Cor. 1. *A mixture of colours may be made which ſhall be like the colour of homogeneal light. But it will differ in this, that the parts of it will be differently refrangible.*

For almoſt any of the priſmatic colours may be imitated by mixing the colours on each ſide. Thus red and yellow make an orange. Orange and green make a ſort of yellow. Yellow and blue make a green. Green and purple make a blueiſh colour. And blue and violet make a purple.

Cor. 2. *Hence one may know how to make a mixture of colours, that ſhall be like the colour of a given object.*

Look at the object thro' a priſm, to ſee the oblong coloured image of it, which divides the colour of it into all the ſimple colours it is compoſed of. Take ſuch a quantity of every ſeparate colour, you ſee in the image, as ſhall be proportional to its intenſity or degree of brightneſs. And ſuch as are faint or hardly appear, may be omitted. Then mix them together.

Cor. 3. *All colours in the world are either the colours of homogeneal light; or compounded of them.*

This is evident from this and the foregoing Propoſitions.

Cor. 4. *A body that reflects no colour, is black.*

Cor. 5. *Hence black is no colour, and white is all colours.* Fig.

PROP. XII.

If any body be placed in ſimple or homogeneal light of any colour; it will appear of the ſame colour.

Exp. 1. Suppoſe the beam of light SF to paſs thro' the hole F, in the window ſhut, and being refracted by the priſm ABC, to caſt the image PT on the oppoſite wall, and to form all the original priſmatic colours. Take a bit of paper, and hold in the red light, and it will appear red. Hold it in the orange, yellow, green, blue, &c. ſucceſſively, and it will appear orange, yellow, green, blue, &c. ſucceſſively. 4.

Take a bit of coloured paper, and hold it in the red light, and it will appear red; alſo hold it in the orange, yellow, green, &c. and it will appear orange, yellow, green, &c. reſpectively. But it will appear moſt luminous when placed in the light of its own kind. The room muſt be dark in trying this experiment, as well as in the following.

Exp. 2. The ſame apparatus continuing. Take *cinnabar*, and place it in the yellow light, and it appears yellow; place it in the blue light and it appears blue, &c. But place it in the red light, and it is moſt luminous, this is its native colour. *Indigo* placed in the red light is red, in the orange it is orange, in the green it is green, &c. and placed in the purple, its own natural light, and it is a moſt reſplendent purple; and its ſplendor gradually diminiſhes, by moving it by degrees thro' the green and yellow to the red. Place a bit of blue ribbon in the red light it appears red; and in the orange, yellow, green, it appears orange, yellow, or green, but faint. Place it in the blue light, and it is a moſt bright blue.

Exp.

Fig. 4. *Exp.* 3. The ſame things remaining. If *cinnabar* and *ultra marine* be held together in the red light; they will both appear red. But the cinnabar will appear of a ſtrong luminous reſplendent red; and the ultra marine of a faint obſcure and dark red. And if they be held together in the blue light, they will both appear blue; but the ultra marine will appear of a ſtrong luminous and reſplendent blue; and the cinnabar of a faint and dark blue. So that the cinnabar reflects the red light much more copiouſly than the ultra marine does; and the ultra marine reflects the blue light much more copiouſly than, the cinnabar doth. The ſame may be tried with other bodies.

Cor. 1. *The colours of all bodies ariſe from hence, that ſome bodies reflect one ſort of rays, others another ſort of rays, more copiouſly than the reſt. And they become coloured, by reflecting the light of their own colours more plentifully, and that of all other colours more ſparingly.*

That this is the only reaſon of their colours, appears from theſe experiments; and further from this conſideration, that the colour of homogeneal light cannot be changed by the reflexion of natural bodies. And therefore they cannot appear coloured by any other means than by reflecting the light of their own colour.

Cor. 2. *'Tis light only that gives colour to bodies; and they have naturally no colour in themſelves.*

For a body cannot be ſeen in the dark, when there is no light. For there is nothing in the body but a diſpoſition to reflect this or that ſort of rays, which makes them be ſaid to be of that colour. And the rays properly ſpeaking are not endued with colours; for in them there is nothing but a certain power or diſpoſition to excite a ſenſation of this or that colour in the brain.

Cor. 3. *Liquors that reflect any colour, tranſmit the other ſorts.* Fig. 4.

For inſtance, a liquor that reflects the blue light, imbibes the red, which will paſs thro' it, if the veſſel is not too broad. And if there be two liquors a red and a blue; tho' either liquor be tranſparent apart, yet you'll not be able to ſee thro' both together. For if only the red rays paſs thro' one, and only the blue thro' the other; no rays can paſs thro' both. Now if bodies become coloured by reflecting, or tranſmitting this or that ſort of rays more copiouſly than the reſt; it muſt be conceived, that they ſtop and ſtifle within themſelves the rays which they do not reflect or tranſmit.

Colour of liquors varies with their thickneſs. Thus a red liquor in a conical glaſs, looks of a pale yellow at bottom; a little higher it grows orange; higher ſtill it becomes red; and where the red is deepeſt, the glaſs is broadeſt. And higher it will tranſmit no rays; they being all loſt by degrees in the body of the liquor. And thence it appears that red is the ſtrongeſt light, and will penetrate fartheſt into a fluid. And this is evident by the obſervations of thoſe that dive into the ſea; for the objects look the redder the deeper one goes; the other rays being reflected back.

Cor. 4. *Hence the red rays are the ſtrongeſt, are leaſt turned out of their way, and penetrate fartheſt into a reſiſting medium. And the reſt of the colours are weaker and weaker in their order; the violet being the weakeſt.*

Cor. 5. *We can make a body to be of any colour we pleaſe, by holding it in the light of that colour.*

PROP.

Fig.

PROP. XIII.

The colours of natural bodies depend upon the ſize and denſity of the compounding particles.

12. *Exper.* 1. Take two object glaſſes belonging to two long teleſcopes, one a plano convex, the other a double convex. Lay the plane ſide of one AEB, upon the convex ſide of the other CED. Then looking perpendicularly upon them, you'll ſee a black ſpot in the middle where they touch; and ſeveral concentric rings of colours about it, at Z, F, G, H, I, A, &c. Theſe rings may be reckoned of ſeveral orders, by reaſon of the ſame colour coming over again. Theſe orders are as follows, 1. Z, black, blue, white, yellow, red. 2. F, violet, blue, green, yellow, red. 3. G, purple, blue, green, yellow, red. 4. H, green, red. 5. I, greeniſh blue, red, &c. Which colours grow weaker and weaker from the center; and alſo narrower and narrower.

But if the light be made to paſs thro' the glaſſes to the eye, the ſpot in the middle will be white; and the orders of colours, going from the center, quite contrary to the others. 1. White, yellowiſh red. 2. Black, violet, blue, white, yellow, red. 3. Violet, blue, green, yellow, red, &c. The black lines between the glaſſes, at, F, G, H, I, A, are diſtances in arithmetical progreſſion, being the places where the ſame colour returns, by reflected light, as at the firſt trial.

Meaſuring the diameters of the rings in the brighteſt part, their ſquares are found to be in arithmetical progreſſion of the odd numbers 1, 3, 5, 7, 9, 11, &c. And ſince one of the glaſſes was plane and the other ſpherical; their diſtances at theſe rings, that is, at Z, F, G, H, &c. muſt be in the ſame progreſſion. Alſo meaſuring the diameters of the faint rings, in the middle between the bright ones; their ſquares, and conſequently the diſtances of the glaſſes there, appear to be in the arithmetical progreſſion of the even numbers, 2, 4, 6, 8, 10, 12, &c. And from the known radius of the lower glaſs, the diameter of any ring may be computed, and conſequently the diſtance of the glaſſes in that place. And hence the diſtance of the glaſſes in the middle of the firſt dark ring, was found to be $\frac{1}{89000}$ part of an inch. And half of it multiplied by 1, 3, 5, 7, &c. gives the diſtance of the glaſſes or the thickneſs of the included air, at the moſt luminous parts of all the bright rings, $\frac{1}{178000}$, $\frac{3}{178000}$, $\frac{5}{178000}$, $\frac{7}{178000}$, &c. and $\frac{2}{178000}$, $\frac{4}{178000}$, $\frac{6}{178000}$, &c. the thickneſs at the darkeſt parts. Fig. 12.

Exp. 2. Wetting the objects with water; as the water crept in between them, the circular rings grew leſs. And by meaſuring their diameters, and from thence finding the diſtances of the glaſſes, theſe diſtances are found leſs than before in proportion as 3 to 4; that is, as the ſines of refraction out of water into air.

Exp. 3. Viewing theſe rings of colours in a dark room, in homogeneal light, they become more diſtinct. And trying all the priſmatic colours ſucceſſively, the red circles will appear to be bigger than the blue or violet. The diſtances of the glaſſes, at the red and violet circles, are as $14\frac{1}{4}$ to 9. Theſe rings were not now of various colours, but appeared only of that priſmatic colour, with which they were illuminated. And the light which fell on the deep ſpaces between the coloured rings, was tranſmitted

Fig. 12. mitted thro' the glaſſes without any variation of colour. From hence the origin of theſe rings is manifeſt; viz. That the air between the glaſſes, according to its various thickneſs, is diſpoſed in ſome places to reflect, and in others to tranſmit the light of any one colour; and in the ſame place, to reflect that of one, where it tranſmits that of another.

The diameter of the 6th circle made by the yellow was $\frac{58}{100}$ of an inch.

Exp. 4. If a little ſoap be agitated in water till it diſſolve, and a bubble of this water blown up, and covered with a glaſs, to keep it from the air. You'll obſerve the bubble grow thin at the top, by the continual deſcent of the water. The bubble at laſt will grow ſo thin at top, as to tranſmit the light and appear black. Then obſerving the bubble by reflected light from the ſky, the black ſpot will be encompaſſed with the ſame coloured circles, and in the ſame order, as in the firſt experiment. As the water deſcends, and the bubble grows thinner, the rings continually grow bigger till the bubble burſts.

If the bubble be viewed by light tranſmitted thro' it, all the colours will be contrary. Therefore it appears that altering the thickneſs of a very thin plate, the colour is altered.

Exp. 5. Take very thin plates of iſinglaſs, talk, or muſcovy glaſs, that ſhow the colours; then by wetting theſe plates, the colours remain the ſame as before, only they become more faint and languid, eſpecially when wet on the under ſide. Therefore the thickneſs of the plate to produce any colour, depends only on the denſity of the plate, and not on the denſity of the incloſing medium. But the colours are more vivid as their denſities are different.

Exp. 6. Two object glaſſes being laid on one another as in experiment 1. If a ſpectator looks obliquely at them, the rings of colours will be increaſed in bigneſs. Hence the colour of ſome plates will change by changing the poſition of the eye. Fig.

From all theſe experiments it is plain, that the colours of bodies, depend upon the thickneſs and denſity of the particles that compoſe them.

Cor. 1. *Hence if the denſity or ſize of the particles in the ſurface of a body be changed; the colour is likewiſe changed.*

Cor. 2. *When the thickneſs of the particles of a body is ſuch, that one ſort of light or colour is reflected; another light or other colours will be tranſmitted. And therefore the body will appear of the firſt colour.*

This appears by Exper. 1. where the tranſmitted light is different from the reflected light.

Cor. 3. *The thickneſs of a plate of air, between two glaſſes, which reflects a deep yellow; muſt be* $\frac{1}{178000}$, or $\frac{3}{178000}$, or $\frac{5}{178000}$, &c. *parts of an inch. And that of a plate of water between two glaſſes, will be* $\frac{3}{4}$ *of it.*

This is plain by the 1ſt experiment, for the thickneſs of the plates of air; and by the 2d, for thoſe of water.

Cor. 4. *The thickneſs of any plate reflecting red, is to the thickneſs of the ſame plate reflecting violet; as* 19 *to* 12.

For by Exp. 3. they were as $14\frac{1}{4}$ to 9.

Cor. 5. *Hence the thickneſs of a plate may be ſuch, that it will reflect different colours.*

If r, v, be the thickneſſes of plates that reflect red and violet. Then (Cor. 4.) $12r$ and $19v$ will be thickneſſes of plates that reflect both.

Fig. Cor. 6. *The parts of bodies and their pores, muſt not be leſs than a certain bigneſs to make them coloured.*

For if a particle be thinner than $\frac{1}{178000}$ part of an inch, it can reflect no yellow (by Cor. 3.) and the like for others. In which caſes the light will be tranſmitted, and the body be tranſparent.

Cor. 7. *The particles of bodies reflect rays of one colour, and tranſmit thoſe of another. And this is the ground of all their colours.*

This appears from experiment the firſt.

Cor. 8. *The bigneſs of the compounding particles of natural bodies may be conjectured by their colours.*

We are let into this by Exp. 1. And from hence a table may be made of the ſeveral orders of colours, to form our judgment by. Such a table you'll find in Sir J. Newton's Optics, pag. 206.

Cor. 9. *The cauſe of reflection, is not the impinging of light on the ſolid parts of bodies. But it is done by a certain power, uniformly diffuſed over the whole ſurface of the reflecting body.*

For in paſſing of light out of glaſs into air, there is a reflexion as ſtrong as out of air into glaſs, or ſtronger; and far ſtronger out of glaſs into water. And air is not likely to have ſtronger reflecting parts than water, or glaſs. And the reflexion is as ſtrong or ſtronger in vacuo.

Again, if light paſſing out of glaſs into air, fall at a greater angle than 40 or 41 degrees, it is all reflected, if more perpendicular it is tranſmitted. Now it is not to be imagined, that light at one angle of incidence ſhould meet with pores enow to tranſmit it; and at another incidence little different, ſhould meet with nothing but parts to reflect it.

Likewiſe in the paſſing of light out of glaſs into air; at a certain incidence the blue is reflected, and the red tranſmitted. Fig.

Again, where the glaſſes touch, there is no ſenſible reflexion.

Likewiſe at particular thickneſs of plates, which are in arithmetic progreſſion, there is no reflection but a tranſmiſſion.

Laſtly the ſmall protuberances, left after poliſhing, being innumerable and infinitely ſmall; would throw the light in all directions. All theſe things ſhew, that the reflexion is not cauſed by impinging.

Cor. 10. *Bodies reflect and refract light by one and the ſame power.*

For when light goes out of glaſs into air, as oblique as it can; if its incidence be made more oblique, it is totally reflected.

Again experience ſhews, that light is alternately reflected and tranſmitted, by thin plates of glaſs for many ſucceſſions, according to the thickneſs of the glaſs.

Likewiſe thoſe bodies that have the greateſt refracting power, reflect the greateſt quantity of light.

PROP. XIV.

Every ray of light in its paſſage thro' any refracting ſurface, is put into a certain tranſient conſtitution or ſtate, which in the progreſs of the ray returns at equal intervals; and diſpoſes the ray at every return, to be tranſmitted, and between the returns to be reflected by it.

This is manifeſt by Exper. 1, 2, and 3d. By theſe experiments it appears, that one and the ſame ſort of rays at equal angles of incidence, on any thin tranſparent plate, is alternately reflected and tranſ-

Fig. Cor. 6. *The parts of bodies and their pores, muſt not be leſs than a certain bigneſs to make them coloured.*

For if a particle be thinner than $\frac{1}{178000}$ part of an inch, it can reflect no yellow (by Cor. 3.) and the like for others. In which caſes the light will be tranſmitted, and the body be tranſparent.

Cor. 7. *The particles of bodies reflect rays of one colour, and tranſmit thoſe of another. And this is the ground of all their colours.*

This appears from experiment the firſt.

Cor. 8. *The bigneſs of the compounding particles of natural bodies may be conjectured by their colours.*

We are let into this by Exp. 1. And from hence a table may be made of the ſeveral orders of colours, to form our judgment by. Such a table you'll find in Sir J. Newton's Optics, pag. 206.

Cor. 9. *The cauſe of reflection, is not the impinging of light on the ſolid parts of bodies. But it is done by a certain power, uniformly diffuſed over the whole ſurface of the reflecting body.*

For in paſſing of light out of glaſs into air, there is a reflexion as ſtrong as out of air into glaſs, or ſtronger; and far ſtronger out of glaſs into water. And air is not likely to have ſtronger reflecting parts than water, or glaſs. And the reflexion is as ſtrong or ſtronger in vacuo.

Again, if light paſſing out of glaſs into air, fall at a greater angle than 40 or 41 degrees, it is all reflected, if more perpendicular it is tranſmitted. Now it is not to be imagined, that light at one angle of incidence ſhould meet with pores enow to tranſmit it; and at another incidence little different, ſhould meet with nothing but parts to reflect it.

Likewiſe in the paſſing of light out of glaſs into air; at a certain incidence the blue is reflected, and the red tranſmitted. Fig.

Again, where the glaſſes touch, there is no ſenſible reflexion.

Likewiſe at particular thickneſs of plates, which are in arithmetic progreſſion, there is no reflection but a tranſmiſſion.

Laſtly the ſmall protuberances, left after poliſhing, being innumerable and infinitely ſmall; would throw the light in all directions. All theſe things ſhew, that the reflexion is not cauſed by impinging.

Cor. 10. *Bodies reflect and refract light by one and the ſame power.*

For when light goes out of glaſs into air, as oblique as it can; if its incidence be made more oblique, it is totally reflected.

Again experience ſhews, that light is alternately reflected and tranſmitted, by thin plates of glaſs for many ſucceſſions, according to the thickneſs of the glaſs.

Likewiſe thoſe bodies that have the greateſt refracting power, reflect the greateſt quantity of light.

PROP. XIV.

Every ray of light in its paſſage thro' any refracting ſurface, is put into a certain tranſient conſtitution or ſtate, which in the progreſs of the ray returns at equal intervals; and diſpoſes the ray at every return, to be tranſmitted, and between the returns to be reflected by it.

This is manifeſt by Exper. 1, 2, and 3d. By theſe experiments it appears, that one and the ſame ſort of rays at equal angles of incidence, on any thin tranſparent plate, is alternately reflected and tranſ-

Fig. tranſmitted, according as the plate increaſes in progreſſion of the numbers 0, 1, 2, 3, 4, &c. ſo that at the thickneſs 1, 3, 5, 7, &c. the rays are reflected; and at the thickneſſes 2, 4, 6, 8, &c. they are tranſmitted; and thereby make the rings of light. And this (by Exp. 5.) depends on both the ſurfaces of the glaſs.

Cor. 1. *The reaſon why the ſurfaces of bodies reflect part of the light falling on them, and tranſmit the reſt, is becauſe ſome rays at their incidence, are in fits of eaſy reflexion, and others in fits of eaſy tranſmiſſion: calling the returns of theſe diſpoſitions of the rays, the fits of eaſy reflexion or tranſmiſſion.*

This appears from the ſeveral rings of colours, made by the object glaſſes; and far more of them are viſible thro' a priſm, and always where the plates are of the thickneſs 1, 2, 3, 4, &c. ſo that they return in equal times.

Cor. 2. *Thoſe ſurfaces which refract the rays moſt ſtrongly in fits of eaſy refraction; do reflect them moſt ſtrongly in fits of eaſy reflexion.*

This follows from the foregoing, and Cor. 9 and 10, of the laſt Prop.

Cor. 3. *When rays of a deep yellow paſs perp. out of any medium into air; the interval of the fits of eaſy reflexion are the* $\frac{1}{89000}$ *th part of an inch. And of the ſame length are the fits of eaſy tranſmiſſion.*

From Exp. 1. of Prop. XIII.

Cor. 4. *When a plate is thinner than* $\frac{1}{178000}$, *the rays are all tranſmitted.*

This appears from the black ſpot, Ex. 1. Prop. XIII.

Cor. 5. *The intervals of the fits are ſhorteſt in the* Fig. *moſt refrangible rays.*

By Exp. 3. Prop. XIII.

Cor. 6. *All bodies appear tranſparent, whoſe particles and pores are too ſmall to cauſe any reflexions.*

Cor. 7. *The opacity of bodies ariſes from the multitude of reflexions within them.*

Cor. 8. *The parts of bodies on which their colours depend, are denſer than the medium included in the pores.*

For where the thin body is rarer than the ambient medium, the obliquity of the rays cauſes the colour to change, ſo that at any obliquity all ſorts of rays will be reflected, which cauſe a whiteneſs. But if the thin body be much denſer, the colours are little changed by the obliquity, and ſo the colour will appear.

Cor. 9. *The conſtituent parts of bodies are the greateſt whoſe colours are moſt dilute.*

For then it indicates a higher order of the colours, as being more mixt, according to Exp. 1. Prop. XIII. where the farther you recede from the center, the more dilute and mixt theſe rings are. And a table may be made therefrom in order to judge by. Such a table you have in Sir J. Newton's Optics, pag. 206.

Cor. 10. *A body becomes tranſparent, by filling the pores of it, with a fluid of the ſame denſity with itſelf.*

For all the parts being of equal denſity, there can be no refraction within it, and therefore the rays will go ſtrait forward and paſs thro' it.

PROP. XV.

If a beam of light paſſes into any tranſparent body, as glaſs; part of it will be reflected at its going in, and the reſt tranſmitted to the further ſurface, where ſome is tranſmitted there, and ſome reflected back to the firſt ſurface. And ſo a quantity of light will be continually reflected back and forward within it, for a vaſt number of times till it be all loſt. Whilſt at every point of incidence there is a part tranſmitted into the air.

Take a glaſs ſpeculum, ground concave on one ſide, and convex on the other, and quickſilvered on the convex ſide. And let a beam of light, coming thro' a ſmall hole in the window ſhut in a dark room, fall upon the ſpeculum. Then hold a paper at the center of the glaſs, ſo that the beam of light may paſs thro' a little hole made in the middle of the paper, to the ſpeculum, and ſo be reflected back to the hole. Then you'll obſerve ſeveral concentric rings of colours round the hole; like theſe before in the object glaſſes by tranſmitted light. Theſe rings grow fainter as they grow larger. Meaſuring the diameters of theſe rings, they are in the ſame proportion as thoſe tranſmitted thro' object glaſſes. Viewing them with homogeneal light, they appear more diſtinct, and to a greater number. The red are the largeſt, the violet leaſt, and ſo in order. Placing the eye where the rings appear plaineſt; the ſpeculum will be ſeen tinged all over with waves of colours. From all theſe things, it is plain, theſe rings are made by the light reflected back and forwards within the glaſs; whilſt ſome, being in the fits of eaſy reflexion, and ſome in the fits of eaſy tranſmiſſion, cauſed theſe rings of colours. When the quickſilver

ſilver was rubbed off the glaſs, there would be the ſame rings but fainter; and therefore the quickſilver only increaſes the light of the colours, by increaſing the reflexion. A metal ſpeculum tried, produced no colours; and therefore the colours depend on both ſurfaces of the glaſs, and the thickneſs between them. From what has been ſaid it is evident, that light undergoes many reflexions in ſucceſſion within a body, and many refractions out of it for many viciſſitudes. Fig.

Cor. 1. *The outward ſurfaces of glaſſes not only reflect and refract incident light, in a regular manner; but ſcatters a part of it in all manner of directions.*

That light is ſcattered from glaſſes in all directions; is evident from this, that the glaſſes can be ſeen in all directions; and therefore the light reflected from them, enters the eye, let it be placed where it will.

Cor. 2. *Hence the image of a candle may be ſeen in a glaſs, repeated a great many times; till by the weakneſs of the light it vaniſhes.*

PROP. XVI.

Bodies act upon light at ſmall diſtances, by attraction and repulſion.

Exper. 1. Make a hole with a pin in a thin piece of lead; and placing it in the hole of a window ſhut, let a beam of the ſun's light paſs thro' it, into a dark room. Then will the ſhadows of hairs, threads, pins, ſtraws, &c. be bigger than they would be, if the rays of light paſſed by them in right lines. And it is the ſame thing, whether the ſmall body be encompaſſed with air or water, &c.

Fig. The ſhadows of ſcratches in glaſs, and the veins in it, are broader than they ought to be.

13. Let X be a hair placed in the beam; ADG, BEH, KNQ, LOR, rays of light paſſing by the ſides of it, bent at X, and falling upon the paper GQ. TI, VS, the direct light. The light paſſing neareſt the hair, as at D and N, is the moſt bent, paſſing to G and Q, the fartheſt aſunder; thoſe that are further off, as at E and O, are leſs bent; and ſo on, to TI and VS. Therefore the force acting upon the light is ſtrongeſt at the leaſt diſtance, making the ſhadow GQ; and weaker at greater diſtances.

Exp. 2. The ſhadow of all ſorts of bodies are bordered with three parallel fringes of coloured light; the neareſt to the ſhadow, the brighteſt; and the furtheſt very faint. The order of the colours, reckoning from the innermoſt, are violet, blue, green, yellow, red. So by looking on the ſun thro' a feather or black ribbon, held cloſe to the eye, ſeveral fringes of colours will appear.

Exp. 3. Let the ſun ſhine thro' a hole, a quarter of an inch broad; place a ſheet of paſteboard two or three feet off, blackt over on both ſides, and in the middle of it, a hole $\frac{3}{4}$ of an inch ſquare, for the light to paſs thro'. Behind the paſteboard faſten the blade of a knife very ſharp, to ſtop part of the light, going thro' the hole. The knife and paſteboard are parallel, and both perp. to the rays. Let a part of the light, which paſſes by the knife edge, fall upon a white paper two or three feet diſtant; and there will be made two ſtreams of light; which will appear fairer, if the light paſſes thro' a hole in the paper, and fall on a black cloth behind it. One of theſe ſtreams ſubtends an angle of about twelve degrees; and is perp. to the edge of the knife. And placing ones eye in the ſtream, a line of light will be ſeen upon the edge of the knife;

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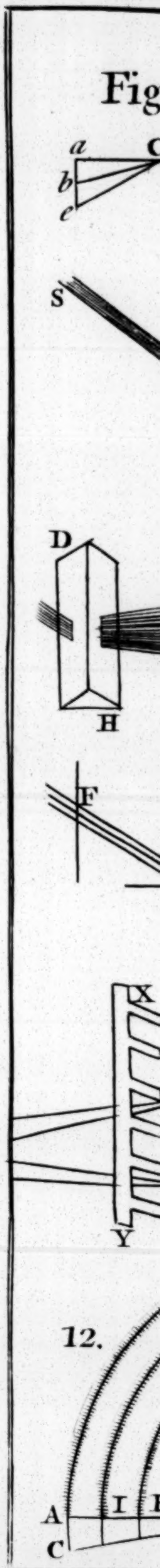

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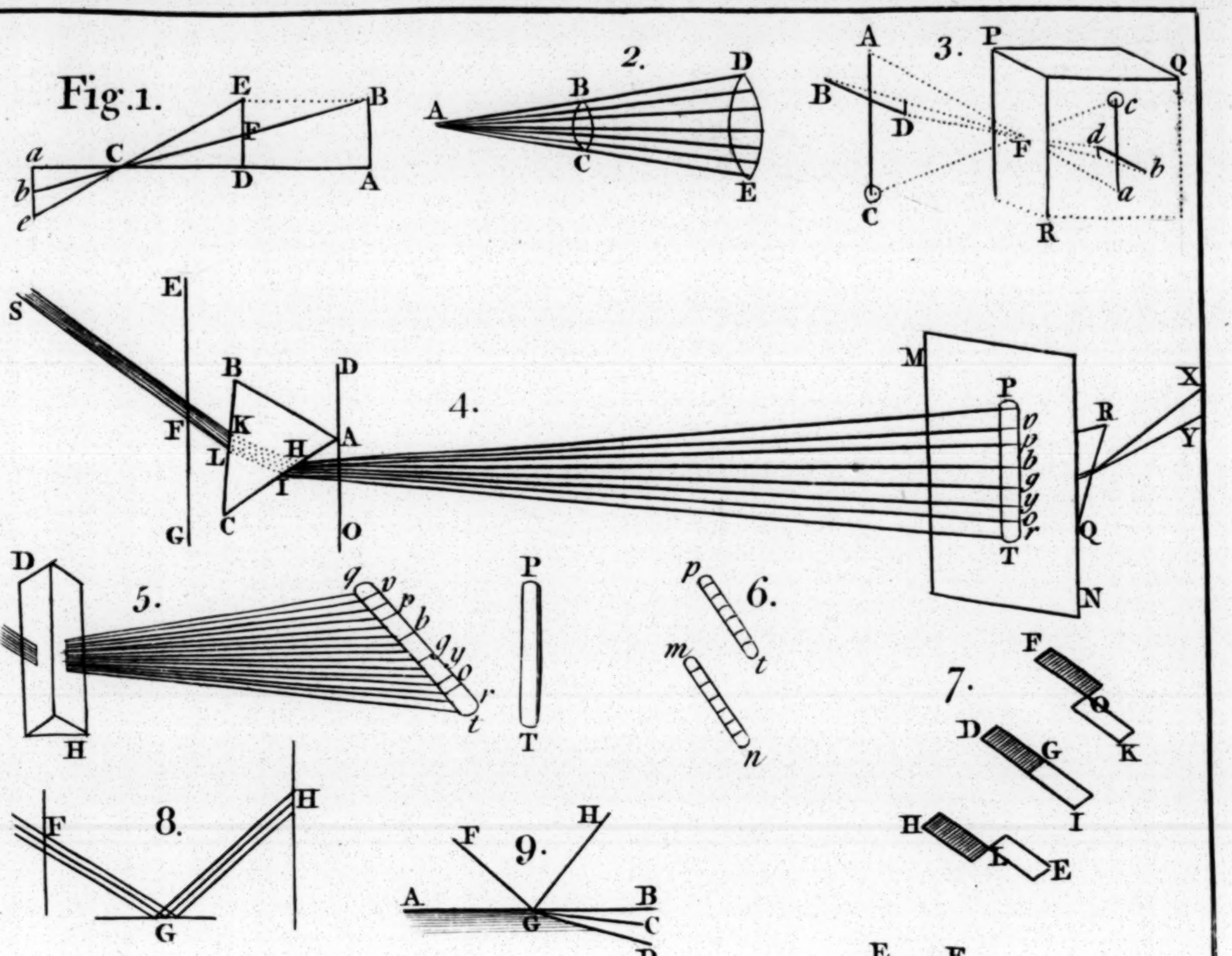

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knife; and even when the eye is placed without the ſtream. Fig. 13.

Exp. 4. Placing another knife, with its edge very near and parallel to the firſt. If they be diſtant the 400th part of an inch, the ſtream of light, paſſing between them, will part in the middle, and leave a ſhadow between them, quite dark. As the knives approach the ſhadow grows broader, and the ſtream narrower at the inner end. Hence the light that is leaſt bent goes to the inner end of the ſtream, and paſſes at the greateſt diſtance from the knives. And this diſtance is about the 800th part of an inch, when the ſhadow begins firſt to appear. And the light which paſſes at diſtances leſs and leſs, is more and more bent; and goes to that hand which is furtheſt from the direct light. A little before the ſhadow appears the fringes begin to appear, on either ſide; and as the knives approach, they grow diſtincter and larger, till they vaniſh, the utmoſt firſt, and the reſt in order.

Hence one may conclude, that the light of the firſt fringe paſſes by the knife, at a greater diſtance than the 800th part of an inch, the ſecond at a greater diſtance, and the third greater ſtill. And the ſtreams of light at a leſs diſtance than any of the fringes.

Exp. 5. Take two knives with ſtrait edges, and prick them into a board, ſo as to make an angle with each other, and faſten them ſo that the angle may be invariable; let the angle be near two degrees. Place theſe in a beam of light, in a dark room, about the 42d part of an inch broad; 10 or 12 feet diſtant from the hole; and let the light paſſing between their edges, fall very obliquely upon a ſmooth white ruler, at $\frac{1}{2}$ an inch or an inch diſtance. Then you'll ſee the fringes made by the two edges of the knives, run along the edges of their ſhadows, in lines parallel to theſe edges, without growing

Fig. ing ſenſibly broader, till they meet in angles equal to that of the knives; and where they meet they end without croſſing. But when the ruler is held at a much greater diſtance, the fringes are narrower, the farther they are from the angular point, where they croſs, and then become much broader.

Hence, the diſtances at which the fringes paſs by the knives, are not increaſed or altered by the approach of the knives. But the angles, in which the rays are bent, are much increaſed by that approach. And the knife which is neareſt any ray, determines which way the ray ſhall be bent; and the other knife increaſes the bent. The diſtance of the edges of the knives (at the meeting of the dark lines between the firſt and ſecond fringes on each ſide) is the 160th part of an inch. Whence half the light paſſes by the edges of each knife, within the 320th part of an inch thereof, and falling upon the paper makes the fringes of the ſhadow of each knife. And the light which falls upon the paper where theſe dark lines meet, paſſes between the knives, where their edges are more than the 160th part of an inch diſtant.

The light which makes the fringes is not the ſame light at all diſtances of the paper from the knives. But when the paper is held near the knives, the light (that makes the fringes) paſſes at a leſs diſtance from the knives, and is more bent, than when the paper is held at a greater diſtance.

Exp. 6. If the paper be held at a great diſtance, perpendicular to the rays; the fringes of the ſhadows will be in the form of hyperbolas, of which there will be ſix for the three fringes on each ſide. In any one of theſe hyberbolas, one aſymptote is parallel to the edge of one knife, and its other aſymptote, is a line drawn perp. to the line which biſects the angle, and paſſes thro' the point where that direct ray of light cuts the paper, which ray Fig. paſſes thro' the angular point of the two knives. Therefore the angle of the aſymptotes of all the hyperbolas will be in that point, where a line drawn from the ſun, thro' the angle of the knives, cuts the paper. And theſe angles will be all obtuſe; being greater than a right angle by half the angle of the knives. And the three hyperbolic fringes on one ſide are equal to theſe on the other. All theſe may be found, by actually meaſuring the ordinates, &c. and ſo proved to be hyperbolas.

Exp. 7. Let the ſun ſhine thro' a ſmall hole made by a pin in a plate of lead; and place a priſm at the hole to refract the light, on the oppoſite wall. The ſhadows of all bodies held in the coloured light, are bordered with fringes of the colour of that light in which they are held; in the red they are red; in the blue, blue, &c. And the fringes made in the red light are largeſt, thoſe in the violet, leaſt; the green ones of a middle bigneſs. And they were ſo at all diſtances from the ſmall body, as a hair, &c.

So then the rays which made the fringes in the red light, paſſed by the hair at a greater diſtance, than thoſe which made the violet fringes. Therefore the hair in cauſing theſe fringes acted alike upon the red rays at a greater diſtance, as upon the violet at a leſs diſtance; and by theſe actions diſpoſed the red light into larger fringes, and the violet into ſmaller, and middle ones into a middle ſize, without changing the colours of the rays.

When a hair is held in a white beam of the ſun's light (Exp. 1 and 2.) and caſts a ſhadow, which is bordered by three fringes of coloured light; theſe colours ariſe from the various inflexions, by which the rays are ſeparated, and being ſeparated, produce each its own colour. In this laſt experiment, where they are ſeparated before the light comes to the hair; the red or leaſt refrangible rays, were inflected at a greater diſtance, and the violet or moſt refrangible rays at a leſs diſtance, making three violet fringes at a leſs diſtance, whilſt the red makes three red fringes at a greater diſtance. And the other mean rays were inflected at mean diſtances, making three fringes of their proper colours, at mean diſtances, from the ſhadow of the hair. And in the white light, theſe colours are ſeparated by the various inflexions of the rays, and their fringes appear all together; and the innermoſt fringes being contiguous make one broad fringe, compoſed of all the colours in due order; the violet being next the ſhadow, and the red fartheſt off, and the reſt in their places. And in like manner the middlemoſt fringes compoſe one broad fringe, of all their colours; and the outmoſt fringes compoſe another broad fringe, like the reſt. And theſe are the three fringes of coloured light, with which the ſhadows of all bodies are bordered, as in Exp. 2.

From all theſe experiments it is obvious, that the rays of light are acted on by ſome force or other that turns them out of their direct road. And as this force bends the rays not into the ſhadow of the body that acts upon them, but from the ſhadow; in that reſpect it muſt be a repulſive force, and (by Exp. 1.) is ſtrongeſt at the leaſt diſtance.

Cor. 1. *It is probable, that the rays of light in the points of contact with bodies, and extremely near it, are acted on by an attractive force. And at greater diſtances with a repulſive one.*

For if the rays were acted on in all caſes and circumſtances by a repulſive force; the light would all be expelled at once out of the luminous body. But it is more probable, that theſe particles of light are kept in the body by ſome attractive force; till they be ſhaken off by the vibrating motion of the Fig. body, and thrown beyond the reach of attraction; the contrary power of repulſion then begins, and drives it away with amazing velocity. Light is acted on entirely by attraction, by refraction in a denſe medium, being made towards the perpendicular.

Cor. 2. *This power which acts upon light is infinitely ſtronger than the force of gravity.*

For the velocity of light being ſurpriſingly great, and the ſmall diſtance the active force extends to, makes the time of acting, ſmall beyond all comprehenſion. And therefore in ſuch an infinitely ſmall time, the finite force of gravity would have no manner of effect.

Cor. 3. *The force of repulſion acting on light, decreaſes very faſt in regard to the diſtance; ſo that in a very ſmall ſpace it becomes nothing.*

The force of gravity decreaſes as the ſquare of the diſtance. But this force being reduced to nothing in a very ſmall ſpace, ſhews that it decreaſes far faſter than in that ratio.

Cor. 4. *Hence bodies and light act mutually upon one another; bodies, upon light by emitting, reflecting, refracting, and inflecting it. And light upon bodies, by heating them, and putting their parts into motion.*

For action and re-action are always equal and contrary.

Cor. 5. *Black bodies, grow hot ſooner by light than any other bodies; and white bodies later.*

For the light falling on them is not reflected, but enters the bodies; and by being often reflected and refracted within it, excite a briſk motion among the internal parts of it. But white bodies reflect moſt of the light, and therefore are but little affected by it.

PROP.

PROP. XVII.

The motion of light is not inſtantaneous, but is performed in time; and ſpends about eight minutes in paſſing from the ſun to the earth.

This is proved from the eclipſes of Jupiter's ſatellites; for when the earth is between the ſun and Jupiter, theſe eclipſes happen about eight minutes ſooner than they ought to do, by the rules of calculation. And when the earth is beyond the ſun, they happen eight minutes later than they ought to do. The reaſon is, the light has a leſs diſtance to move over, from the ſatellites to the earth, in the firſt caſe, than in the ſecond, by the diameter of the earth's orbit. And therefore the radius of the earth's orbit requires eight minutes of time for the light to paſs thro' it.

Cor. *It is very probable, that the rays of light are extremely ſmall bodies, projected with incredible velocity from the luminous body.*

For, 1. It is the property of a body to move thro' ſpaces which are proportional to the times, by a uniform motion; and light does this. 2. It is acted on by a body, when is is reflected, refracted, and inflected; and bodies can only act upon bodies. 3. Light cannot be increaſed or diminiſhed. It can only be increaſed in any place, by robbing other places of its light; and it can only be diminiſhed by conveying it to a different place. And all this may be done with bodies. So that we cannot increaſe it in one place, but we muſt decreaſe it in another. 4. Light conſiſts of parts both ſucceſſive in the ſame lines, and contemporary in ſeveral lines. For in the ſame place one may ſtop that which comes one moment, and let that paſs which comes the next moment. And at the ſame time, one may ſtop it in one place and let it paſs in another. For that part of the light which is ſtopt, cannot be the ſame with that which is let paſs. And all theſe properties agree with what we find in bodies. Fig.

PROP. XVIII.

If light enters into a denſe medium, and in its motion in that medium, a part of it be loſt, ſo that only the part p gets thro' the ſpace a. Then the quantity of light which gets thro' the ſpace na in the ſame continued medium, will be p^n.

I take it for granted, that if any quantity of light paſſes into a denſe medium, and in moving thro' the ſpace a loſes any part, as $\frac{1}{10}$ thereof; if half the quantity of light enters it, it will loſe $\frac{1}{10}$ of that half in the ſame ſpace; or if a quarter of the light enters it, it will loſe $\frac{1}{10}$ of that quarter, and ſo on. Therefore there will always be a certain part of the incident light, get thro' equal ſpaces.

Suppoſe then $\frac{8}{9}$ the light, gets thro' the ſpace a; then $\frac{8}{9}$ of that, or $\frac{8}{9}$ of $\frac{8}{9}$ will get thro' the next ſpace a, that is, $\left.\frac{8}{9}\right|^{2}$ will get thro' $2a$, and $\frac{8}{9}$ of that or $\left.\frac{8}{9}\right|^{3}$ will get thro' $3a$; and ſo on. Therefore $\left.\frac{8}{9}\right|^{n}$ of the entring light will get thro' the ſpace na. And inſtead of $\frac{8}{9}$ putting p, the quantity of light getting thro' the ſpace na, will be p^n.

Cor 1. *If* $1-p$ *be the light loſt or deſtroyed in paſſing thro' the ſpace or depth a;* $1-p^n$ *will be the light loſt in paſſing thro' the depth na.*

Cor. 2. *The light tranſmitted at the depth ma, is to the light tranſmitted at the depth na, as* p^m *to* p^n.

Cor. 3. *The light tranſmitted thro' the depth a, is to the light tranſmitted thro' the depth na, as* 1 *to* p^{n-1}.

For the lights tranſmitted are as p to p^n, that is, as 1 to $\frac{p^n}{p}$ or p^{n-1}; where n may be whole or fractional.

PROP. XIX.

The brightneſs of an object is nearly the ſame, at all diſtances of the eye from it.

For if the eye at any diſtance receive any quantity of light, at half that diſtance it will receive four times the light, and the picture upon the retina will be four times as big, whence the denſity or brightneſs continues the ſame. Alſo at a third part of the diſtance it receives nine times the light, and the picture in the eye will be nine times as big; and therefore the denſity is ſtill the ſame. Or in general at $\frac{1}{n}$ of the diſtance, n times the light comes into the eye, and the picture is n times as big, and the denſity the ſame as before, becauſe n times the light is ſpread over n times the ſpace.

PROP. XX.

The proportion of moon light to day light, or the light of the ſun, at full moon, is as 1 *to* 96 *thouſand, nearly.*

For the moon's radius is to the earth's radius as 1 to 3.65; and ſince the radius of the moon's orbit is 60 of the earth's radii; therefore the radius of the moon's orbit, or its diſtance from the earth is 60 × 3.65 or 219 of the moon's radii.

Now ſuppoſe equal ſpaces at the earth and moon to be equally illuminated; and that the whole ſurface of the moon was thus equally illuminated, and the light reflected therefrom all around, ſo far as the ſurface of a ſphere paſſing thro' the earth, deſcribed from the moon's center; then all this light is ſpread upon this ſurface; and conſequently its denſity will be leſs in the reciprocal proportion of theſe ſurfaces, or reciprocally as the ſquares of the diameters. That is, the denſity of the light at the moon (which is ſuppoſed the ſame with day-light) is to the denſity at the earth (reflected from the moon); as the ſquare of 219 to 1, or as 47961 to 1. But ſince in reality, there is not ſo much light falls on the moon, being no more than what falls on her diſk, and her diſk being but half her hemiſphere; the denſity of light at her ſurface, upon each hemiſphere will be but half as much. And it matters not whether any falls upon the oppoſite diſk or not, for (by the rule of proportion) that would all be reflected to the oppoſite hemiſphere, and does not concern us, who are oppoſed to the full moon. Therefore it will now be, as day-light, to moon-light; ſo is 47961 to $\frac{1}{2}$, or as 95922 to 1, or in round numbers as 96000 to 1.

Cor. 1. *Moon-light is to day-light; as half the ſquare of the moon's radius, to the ſquare of the moon's diſtance, when ſhe is full. And in the quadratures, as $\frac{1}{4}$ the ſquare of the moon's radius, to the ſquare of the moon's diſtance.*

After the ſame manner may be found the light of any other celeſtial body, compared with day-light; let it be *Venus*. Let $d =$ ſun's diſtance from the earth, $v =$ Venus diſtance from the ſun, $a =$ her diſtance from the earth, $r =$ her radius. Then if Venus and the earth were equally illuminated by the ſun, then is her light to day-light : :

Fig. $\frac{1}{4}rr : aa$. But Venus being nearer the ſun, is more enlightned in the ratio of $\frac{dd}{vv}$ to 1; therefore her light is to day-light $:: \frac{1}{4}rr \times \frac{dd}{vv} : aa :: \frac{1}{4}rrdd : vvaa$. But ſince in her quadratures, $d = a$ nearly, therefore, Venus's light is to day-light; as $\frac{1}{4}rr$ to vv. And that is as 1 to 804.000000, nearly. Hence,

Cor. 2. *The light of Venus in her quadratures, is to day-light; as* 1 *to* 800 *millions.*

This is ſuppoſing the radius of Venus, to her diſtance from the ſun, to be as 1 to 14200.

Cor. 3. *Hence the light of the moon is* 8000 *times as great as the light of Venus. Suppoſing they both of them reflect all the light that falls on them.*

BOOK II.

CATOPTRICS; OR REFLECTED VISION.

DEFINITIONS.

DEF. I.

CATOPTRICS, treats about that part of Optics, which relates to ſeeing, or making the pictures of objects by reflected light. Fig.

DEF. II.

A *Speculum* or *Mirror* is any poliſhed ſurface, commonly of glaſs or metal, which reflects the rays of light; and may be either *plain*, *concave*, or *convex*. The glaſs ones are commonly quickſilvered over on the backſide; to make the reflexion ſtronger.

PROP. I.

If a ray of light be reflected from a plain ſurface; the angle of reflexion will be equal to the angle of incidence.

It is ſhewn in Prop. XVI. that the rays of light in being reflected or refracted, are acted on by the bodies they come near, by repelling or attracting them. And theſe forces, be what they will, muſt act in lines perpendicular to the ſurfaces of theſe bodys; and that force muſt be equal at equal diſtances from the reflecting or refracting ſurface.

Let

Fig. 14. Let AB be the incident ray, CD the reflected one. RS the reflecting ſurface, PQRS the infinitely thin ſpace, within which the repulſive force of the body is contained, AB*m*CD the progreſs of the ray. Draw an infinite number of lines parallel to PQ or RS, ſuch as *nt*, *oq*. Then ſince the repulſive force begins at B, it will continually bend or inflect the ray, into the curve B*om*, till it comes to its loweſt point *m*, where it will be parallel to the reflecting ſurface. Afterwards it will recede from the plane SR, deſcribing the curve *mq*C. And ſince the forces are equal at the equal diſtances *o* and *q*, *n* and *t*, &c. therefore the particle of the curve *mq*, will be equal and ſimilar to the particle *om*, and *qt* to *on*. And for the ſame reaſon, all the particles of the curve *mq*C will be equal and ſimilar to the correſponding particles of the curve *mo*B; and therefore the whole *mq*C will be equal and ſimilar to the whole *mo*B. And conſequently the angles at C and B will be equal; and the angle of reflexion equal to the angle of incidence, being their complements.

Cor. 1 *If a ray of light be reflected from any point of a curve ſurface; the angle of reflexion is equal to the angle of incidence.*

For it is the ſame thing as if it was reflected from a plane touching the curve in that point.

Cor. 2. *The ray of incidence and reflexion are both in a plane, perpendicular to the reflecting ſurface, in the point of incidence.*

For there is no force acting out of that plane.

Cor. 3. *A ray of light falling perpendicular to any ſurface, is reflected back in the ſame perpendicular. But moving parallel to it, will continue ſo.*

Cor. 4. *If the reflected ray at* O, *return back thro'* OS, *it will be reflected to* R. Fig. 14.

Scholium.

The curve B*omq*C deſcribed by the ray of light, is ſo extremely ſmall, that it may be looked upon as a ſingle point. And therefore the points B and C may be ſuppoſed to coincide.

PROP. II.

If rays proceeding from a radiant point R *fall upon a reflecting plane ſurface* PL; *the focus of the reflected rays, will be at* F, *in the line* RF *perp. to the reflecting ſurface; and as far on the other ſide of it. That is* $PF = PR$. 15.

For ſince RPF is perp. to PL; let RS be any incident ray; and SO the reflected ray. Produce OS backward to F. Then (Prop. I.) $\angle PSR = \angle LSO = PSF$. Therefore in the right angled triangles PSR, PSF; all the angles are reſpectively equal, and PS common; therefore RF equal PF. Therefore the reflected ray SO proceeds from F, in the perp. RF, ſo that $PF = PR$. And by the ſame reaſoning, if RN be any other incident ray, its reflected ray NH will proceed from F. And therefore F is the virtual focus of all the reflected rays, which therefore all diverge from F.

Cor. 1. *The diſtance of the focus* F *from the eye at* O, *is equal to the incident and reflected rays taken together*, $OF = OS + SR$.

For $OF = OS + SF = OS + SR$.

Cor. 2. *Rays converging to* F, *and reflected by the plane* PL, *converge to* R.

Cor. 3. *If rays proceeding from an object at* R *be reflected by the plane* PL *to an eye at* O; *the object*

Fig. 15. *will appear to be placed at* F, *the focus of the reflected rays, where they laſt diverged from.*

For we always judge an object to be in that place, from whence the rays proceed directly to the eye; which here is from F.

PROP. III.

16. 17. *If rays proceeding from a radiant point* R, *fall upon any point* S *of a reflecting ſpherical ſurface* VS, *and* F *be the focus of the reflected rays; it will be,* SR : CR : : SF : CF. C *being the center of the ſphere.*

For (*fig.* 16.) the angle of reflexion CSF being equal to the angle of incidence CSR (Prop. I.); therefore the angle RSF is biſected; and therefore (Geom. II. 25.) SR : SF : : CR : CF.

And (*fig.* 17.) draw FA parallel to RS, and produce CS to B; and if FSD be the reflected ray, then the angle FSA = BSD = (Prop. I.) BSR = SAF; therefore SF = AF. Then by the ſimilar triangles CRS and CFA, SR : CR : : AF or SF : CF.

Cor. 1. *If the radiating point* R, *and the focus* F, *be both on one ſide the reflecting ſurface. If the rays diverge from one, they converge to the other.*

Cor. 2. *If the radiating point* R *and focus* F *be on different ſides of the reflecting ſurface; if the rays diverge from one, they diverge from the other. But if they converge to one, they converge to the other, by reflection.*

Cor. 3. *If* R *is at an infinite diſtance, then* SF = CF.

For then CR = SR, and conſequently CF = SF.

Cor. 4. *If* R *is the radiant point*; *make the* $<$ Fig.
$RSC = CSF$, *or* $RSB = BSD$, *and draw* SF, *or* 16.
DSF, *and* F *is the focus of the reflected rays.* 17.

Cor. 5. *The focus of rays proceeding from any radiant point* R; *is in the line* RCV, *joining the point* R *and the center.*

For taking $Vs = VS$, the focus is in the place where the rays meet, which are reflected from all points of the arch S*s*. Let RS, R*s* be two correſpondent ones, then the angles SRF, *s*RF, being equal, and alſo RSF and R*s*F; therefore SF, *s*F will meet ſomewhere in the line RCV, at F.

Cor. 6. *A ſmall object placed in the center, cannot be ſeen by reflexion. Or the eye placed in the center, can ſee nothing by reflexion.*

For all the reflected rays go alſo to the center.

PROP. IV.

If VA *be a reflecting ſpherical ſurface, whoſe center is* C. *And if a ray of light* RA (*or* GA) *parallel to* CV, *fall upon* A, *and be reflected to* F. *Draw the tangent* AT; *then will* $TF = FC$. 18.

For CAT being a right angle, and (Prop. I.) $<$ CAF being $=$ angle CAR; we ſhall have

$$CAT = \frac{RAF + FAG}{2},$$

and

$$CAF = \frac{RAF}{2}$$

then by ſubtraction $FAT = \frac{FAG}{2} = TAG = ATF$;

therefore $TF = AF = FC$, becauſe $< CAF = CAR = ACF$.

Cor. 1. *Hence* FC, FA, FT, *are all equal to one another.*

Cor. 2. *The tangent* AT *biſects the angle* FAG.

Fig. Cor. 3. *If* CQ = QV, *then* Q *is the focus of the*
18. *rays falling on* V, *or the principal focus.*

Cor. 4. *And* QF (= *half* TV) *is the ſpace on which all the rays fall that come from the whole ſurface* AV.

19. Cor. 5. *If* AV *be any curve, and a ray of light* RA (*or* GA) *be reflected into the line* AH. *If* VL *be drawn parallel to* AR, *and* AL *perp. to the curve at* A. *Then* AH = HL.

For produce AL to C, that AC may be the radius of curvature in A; draw CF parallel to VL, and produce AH to interſect it in F. Then F is the focus of the rays reflected by the circle, paſſing thro' A, and therefore of the portion of the curve at A that coincides with it. Therefore (Cor. 1.) CF = AF, and conſequently (by ſimilar triangles) LH = AH.

Cor. 6. *When parallel rays fall on the ſurface of a ſphere, thoſe neareſt the vertex, have their focus neareſt the center.*

PROP. V.

20. *If* aVA *be a ſmall portion of a reflecting ſpherical*
21. *ſurface,* C *its center,* V *its vertex;* R *the radiant point, from or towards which the rays flow;* F *the focus after reflexion at* A. *Then it will be* VR : VF : : CR : CF, *nearly.*

For (Prop. III.) If A be the point of incidence, it is AR : AF : : CR : CF. But the arch VA being extremely ſmall, AR = VR, and AF = VF, very near. Therefore VR : VF : : CR : CF.

Cor. *The points* R, F *are conjugate foci to one another; that is, if* R *be the radiant point,* F *will be the focus; or if* F *be the radiant point,* R *will be the focus. Where I uſe radiant points in a general ſenſe,*

for the point, which the rays either diverge from, or Fig.
converge to, before reflexion.

PROP. VI.

22. *If a*VA *be a ſmall portion of a reflecting ſpherical ſurface, whoſe center is* C, *and* V *its vertex. And if parallel rays, or thoſe from an infinite diſtance, as* RA *or* GA, *fall upon it, and* F *be the focus of the reflected rays; then* F *is in the middle of the radius* VC, *or* VF = FC.

For (by the laſt Prop.) VR : CR : : VF : CF. But becauſe R is infinitely diſtant, VR = CR, therefore VF = CF.

Cor. 1. *Rays diverging from* F, *the middle point of the radius, or converging to* F, *and reflected by the ſpherical ſurface* VA: *will after reflection go parallel to one another.*

Cor. 2. *A luminous body placed in* F *the middle point of the radius, will caſt a light to a great diſtance, by reflexion from the concave ſurface.*

Cor. 3. *The principal focus of a reflecting ſpherical ſurface, is in the middle point of the radius at* F. *And the principal focal diſtance* FV, *is half the radius.*

PROP. VII.

23. 24. *Let* A*a be a ſmall part of a reflecting ſpherical ſurface;* C *the center,* V *the vertex,* P *the principal focus,* R *the radiating point (or that which the rays diverge from, or converge to, before reflexion). And if* F *be the focus after reflexion, then will* PR, PC *or* PV, *and* PF, *be in continual proportion.*

For (*fig.* 23), ſince VF — PF = PF + FC, therefore VF — FC = 2PF. But (Prop. V.) VR

Fig. : VF : : CR : CF ; by diviſion VR : VF : : VR —
23. CR (VC) : VF — CF (2PF) : : 2VP : 2PF : : VP :
24. PF. And again, by diviſion VR : VF : : VR — VP (PR) : VF — PF (VP). Therefore PR : VP : : VP : PF.

And (*fig.* 24.) VR : VF : : CR — VR (CV or 2VP) : CF — VF (2PF) : : VP : PF. And by addition, VR : VF : : VR + VP (PR) : VF + PF (VP.) Therefore PR : VP : : VP : PF.

Cor. 1. F *and* R *are conjugate foci to one another, and lye always the ſame way from* P.

Cor. 2. *The rectangle of the diſtances of the radiant point, and of the focus, from the principal focus; is equal to* $\frac{1}{4}$ *the ſquare of the radius of the ſphere.* $PR \times PF = \frac{1}{4} VC^2$.

Cor. 3. *The principal focal diſtance, is equal to the rectangle of the diſtances, of the focus, and of the radiant point, from the vertex, divided by the ſum or difference of theſe diſtances; according as* F *and* R *are on the ſame or different ſides of* V. $VP = \frac{VR \times VF}{VR \pm VF}$.

For we had VR : VF : : PR : PV. And $VR \times PV = VF \times PR = VF \times \overline{VR \mp VP}$, and $PV \times \overline{VR \pm VF} = VF \times VR$.

Cor. 4. *If* R *be the radiant point,* F *its correſpondent focus; or* R *and* F, *conjugate foci; the diſtance of that focus* F, *from the principal focus* P, *or* $PF = \frac{VF^2}{VR \pm VF}$: *according as* F *and* R *are on the ſame or different ſides of* V.

For $PF = VF - VP = VF - \frac{VR \times VF}{VR + VF} = \frac{VF^2}{VR + VF}$. Or $PF = VP - VF = \frac{VR \times VF}{VR - VF} - VF = \frac{VF^2}{VR - VF}$.

SCHOLIUM.

Fig. 23. 24. The 3 laſt propoſitions lay the foundation of all that is practicable in Optics, ſo far as reflected light is concerned. For no curve whatever is made uſe of to reflect the rays of light, except the ſphere; no method being yet found out to grind glaſſes accurately into any other curve. And in uſing ſpherical ſurfaces, a ſmall ſegment only is to be taken; becauſe the parts of the ſurface remote from the vertex, reflect the rays to different focus's; which are ſo much further from the principal focus, as the reflecting part is further from the vertex. And therefore if a large ſegment was uſed, the rays from any ſingle point of an object, could not be brought to a ſingle point in the focus, which would be attended with a deal of confuſion.

The following propoſition gives a general rule for determining the foci of all glaſſes univerſally whether convex or concave; and whether the rays diverge or converge, and from whatever diſtances, finite or infinite, they proceed.

PROP. VIII. *Prob.*

25. *Let* A*a be a ſmall portion of a reflecting ſpherical ſurface,* C *its center,* R *the radiating point. To find* F *the focus of the rays after reflexion.*

Put the diſtance VR $= d$
radius of the ſphere CV $= r$
focal diſtance VF $= f$.

Then $f = \frac{dr}{2d + r}$, *very near. Where, if the rays converge, or* AV *be concave,* d *or* r *will be negative.*

For CR $= d + r$, and CF $= r - f$. And (by Prop. V.) VR : VF : : CR : CF; that is, $d : f : :$ $d + r$

Fig. $d + r : r - f$. Whence $dr - df = df + rf$, and
25. $dr = 2df + rf$; and reduced, $f = \frac{dr}{2d+r}$.

Cor. 1. *If d is infinite, or* R *is at an infinite diſtance, then r vaniſhes, and $f = \frac{1}{2}r$.*

Cor. 2. *If* R *is on the concave ſide, d is negative, and $f = \frac{-dr}{-2d+r}$, that is, $f = \frac{dr}{2d-r}$. And moreover, if r is infinite, as in a plane ſurface, $f = -d$.*

Cor. 3. *If r is negative, or the reflecting ſurface concave towards* R, *then $f = \frac{-dr}{2d-r}$, that is, $f = \frac{dr}{r-2d}$.*

Cor. 4. *If* AV *is concave towards* R, *and d infinite, then r is negative, and $f = \frac{-dr}{2d}$, that is, $f = -\frac{1}{2}r$.*

Cor. 5. *If it is a plane ſurface, then r is infinite, and $f = d$.*

And many other corollaries may be deduced.

SCHOLIUM.

It is eaſy to know whether the focus be real or virtual, by the direction and motion of the rays; that is, by their converging or diverging after reflexion.

From this general ſolution it is plain, that when the focal diſtance comes out affirmative, it is to be taken right forward; but if negative, it muſt be taken backward, from the vertex.

From this Prop. ſeveral other Problems may be reſolved. As if it was required to know, what muſt be the radius of a ſphere, to repreſent the given object R, at the given focus F. Here r is to be found from the equation $f = \frac{dr}{2d+r}$; multiply by $2d+r$ and we have $2df + rf = dr$, and $dr - fr = 2df$, whence $r = \frac{2df}{d-f}$, the radius of the ſphere. From this equation, it is plain, that if d be greater than f, the reflector will be convex towards the object. But if f be greater than d, r will come out negative, and the reflector will be concave to the object. If d be equal to f, r comes out infinite, and in this caſe, the ſpeculum muſt be a plane ſurface. Fig. 25.

Again, if it be required to find the diſtance of an object R, from the given ſpeculum AV, to have the focus at the given place F. Here we want to find d, from the equation $f = \frac{dr}{2d+r}$; by multiplying, we have $2df + rf = dr$, and tranſpoſing, $dr - 2df = rf$, whence $d = \frac{rf}{r-2f}$, the diſtance from a convex ſpeculum. But if the ſpeculum be concave to the object, then $d = \frac{rf}{r+2f}$. Hence in a convex ſpeculum if r be greater than $2f$, the object will be on the convex ſide. But if $2f$ be greater than r, it will be on the concave ſide. And if $r = 2f$, the diſtance of the object will be infinite; and f will be the focal diſtance of parallel rays.

The principal focus of a reflecting ſpherical ſurface may alſo be found by experiments, as follows.

1. For a concave reflector. Hold it directly before the ſun, and hold a bit of paper before it to receive the reflected light. Move this paper back and forward till the white ſpot of light be the leaſt poſſible, and that is the focus.

Or,

Fig. 25. *Or thus*, cover it with paper having ſeveral pin holes made in it. Then hold a piece of paper before it, and you'll ſee ſo many white ſpots on it made by the light reflected thro' the pin holes. Move the paper back and forward, till they all coincide, and there is the focus.

2. For a convex reflector. Cover it with paper, having two pin holes made in it on contrary ſides. Then expoſe it to the ſun beams, and hold another paper before it, having a hole made in it ſo big as to let the ſun's rays paſs thro' to the two pin holes. Then you will ſee two white ſpots of reflected light on each ſide the hole. Move the paper back and forwards, till the diſtance of the ſpots be twice the diſtance of the holes in the cover, firſt marked upon the paper. Then the diſtance of the paper from the lens is the principal focal diſtance.

PROP. IX.

26. *If rays flow from one focus* R *of a ſpheroid or hyperboloid* BA; *they will be reflected by the curve ſurface, into the other focus* F.

For (Conics, B. I. Prop. X.) the lines AR, AF drawn to the foci of an ellipſis, make equal angles with the curve; and therefore the angle of incidence is equal to the angle of reflexion. And the ſame is true of the hyperbola (ib. B. II. Prop. IX.); therefore if RA be the incident ray, AF will be the reflected one.

Cor. *Hence* F *is the real focus in the ellipſis, and only a virtual focus in the hyperbola.*

PROP. X.

If rays RA, *parallel to the axis of a paraboloid* BA, *be reflected by the ſurface thereof; they will all be reflected into* F *the focus of it.* 27.

For (by Prop. VIII. Cor. 3. B. III. Conics), the line FA drawn to the focus, and RA parallel to the axis, make equal angles with the curve at A. Therefore if RA be the incident ray, AF will be the reflected one.

Cor. *Rays proceeding from the focus* F, *will be reflected by the ſurface of the paraboloid, parallel to the axis.*

PROP. XI.

Wherever the rays, which come from all the points of an object, meet again in ſo many other points, after they have been made to converge by reflexion; there they will make the picture of the object upon a white paper, or any white body.

For if PR be any object, A*a* a concave ſpeculum, the rays that diverge from any point Q, fall upon the ſpeculum A*a*, and are reflected to their focus *q*, where they all meet, and upon a white paper held there give *q* the image of Q. In like manner the light which diverges from P, will be reflected from the ſpeculum to their focus *p*. And all the light diverging from R, will be reflected to their focus *r*. And thus from every point in the object, the light will be reflected to ſo many correſpondent points in the focus; and will form there the picture *pqr*, of the object PQR, in its proper ſhape and colours; but ſometimes the picture will be inverted; as in this figure, where the upper end is downward, and the right to the left hand. 28.

Cor.

Fig. 28. Cor. 1. *The rays of light flowing from all the points of an object* PR, *paint the image thereof pr in the real focus q; and it may be ſeen there hanging in the air, by an eye at* O, *placed at a proper diſtance, and ſituation to receive the rays coming from it.*

Cor. 2. *The extreme point of the image, as r, is determined by the extreme ray* R*r drawn thro' the center* C.

PROP. XII.

The image of a right line made by a reflecting plane ſurface, is alſo a right line.

29. Let AB be a reflecting plane, PR a right line, Q the middle point of it. Draw P*p*, Q*q*, R*r*, perpendicular to the plane AB, cutting it in D, I, L. And make D*p* = DP, I*q* = IQ, and L*r* = LR, then (Prop. II.) *p*, *q*, *r*, will be the foci of P, Q, R; and *pr* the image of PR. Whence the figure DPQRL is ſimilar and equal to the figure D*pqr*L, by conſtruction. And therefore ſince PQR is a right line, *pqr* is alſo a right line.

Cor. 1. *Hence the image of a plain figure, ſeen in a reflecting plain ſurface, is alſo a plain figure, equal and ſimilar to the figure itſelf.*

For all the right lines drawn in it, are repreſented by right lines, equal to thoſe in the expoſed figure. And therefore the figure and its image are equal and ſimilar.

30. Cor. 2. *The image of any object, made by an upright reflecting plane, has its parts in a contrary poſition, the left ſide being to the right, and the right to the left hand; the difference being the ſame as the ſeal, and its print on the wax. But in a horizontal reflector, the object will appear upſide down; the parts lying to either hand, remaining the ſame as in the object.*

For if ABC be a perp. reflector; the object PQR, making the image pqr, where P on the left being neareſt the glaſs, will be repreſented by p on the right hand, alſo neareſt the glaſs. But ſtanding at D and viewing the upright object PQR, in the reflector ABC, laid horizontally; the loweſt point P, neareſt the glaſs, will be repreſented by the higheſt point p, alſo neareſt the glaſs; whilſt R *and* r are on the right, and Q, q on the left. Fig. 30.

Cor. 3. *What has been ſaid of the image of an object, is alſo true of the image of an image.*

Cor. 4. *Changing the ſituation of the eye, does not at all change the ſituation of the image; it will always appear in the ſame place, whilſt the object and reflector are fixt.*

Cor. 5. *When an object is ſeen by a plain ſpeculum; its image appears as far behind the ſpeculum, as the object is before it.*

PROP. XIII.

If AVa *be a ſpherical reflector, the object* PQR *an arch of a circle, having the ſame center* C, *its image* pqr, *by reflexion, will alſo be an arch of a circle, having the ſame center* C. *And its image will be direct or inverted, according as it is on the ſame or different ſide of the center* C, *with the object.* 31. 32.

Here p is the focus of P, q of Q, and r of R. Let e, f, g, be the principal foci, of the points A, V, a, of the reflecting ſurface. Then (Prop. VII.) we have fQ : fC (fV : fq,) in continual proportion, and gR : gC : gr; alſo eP : eC : ep, alſo in continual proportion. But the two firſt terms are the ſame in all, and therefore the laſt; that is $fq = gr = ep$. And ſince $f\mathrm{C} = g\mathrm{C} = e\mathrm{C}$, the ſums or

Fig. remainders will be equal, $qC = rC = pC$, therefore
31. rpq is the arch of a circle whoſe center is C.
32. And ſince the rays croſs at the center C, if the image be on the ſame ſide of C as the object, it will be in the ſame poſition; but on different ſides, in a different poſition.

Cor. 1. *The length of the object* PR, *is to th length of the image* pr; *as* CP *the object's diſtanc from the center, is to* Cp *the diſtance of the image from the center.*

Cor. 2. *On the contrary, if* pqr *be a circular object, its image* PQR *will alſo be a circular arch concentri with the object.*

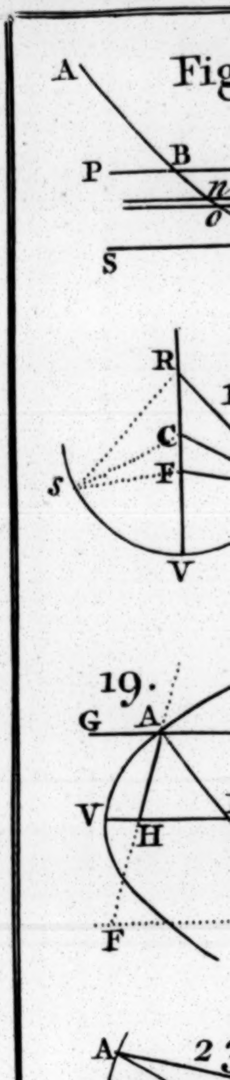
Cor. 3. *The image of a ſpherical ſurface, ſeen by reflexion in a ſpherical reflector parallel to it, will ap-pear a ſpherical ſurface, alſo parallel to it.*

PROP. XIV.

33. *If a right line be placed directly before a concav*
34. *reflector, its image will appear concave; but in a con-vex reflector, the image will appear convex.*

To the center of the glaſs C, deſcribe the arches PQR, pqr; ſo as pqr may be the image of PQR. And let the expoſed lines be OQ or oq, tangents to the arches PQ, pq, in Q and q. Then if the arch PQ be an object, the arch pq will be its image. Let e, f, g, be the principal foci to the points A, V, a. Then (*fig.* 33.) we ſhall have (by Prop. VII.) $ep : eC : eP$, and alſo $eo : eC : eO$, continua proportionals; ſuppoſing o the image of O. There-fore $ep \times eP = eC^2 = eo \times eO$, whence $ep : eo :$ $eO : eP$. And by diviſion $ep : ep - eo\ (op) : : eO$ $eO - eP\ (OP)$, and alternately $ep : eO : : op : OP$, o

$\frac{ep}{eO}$



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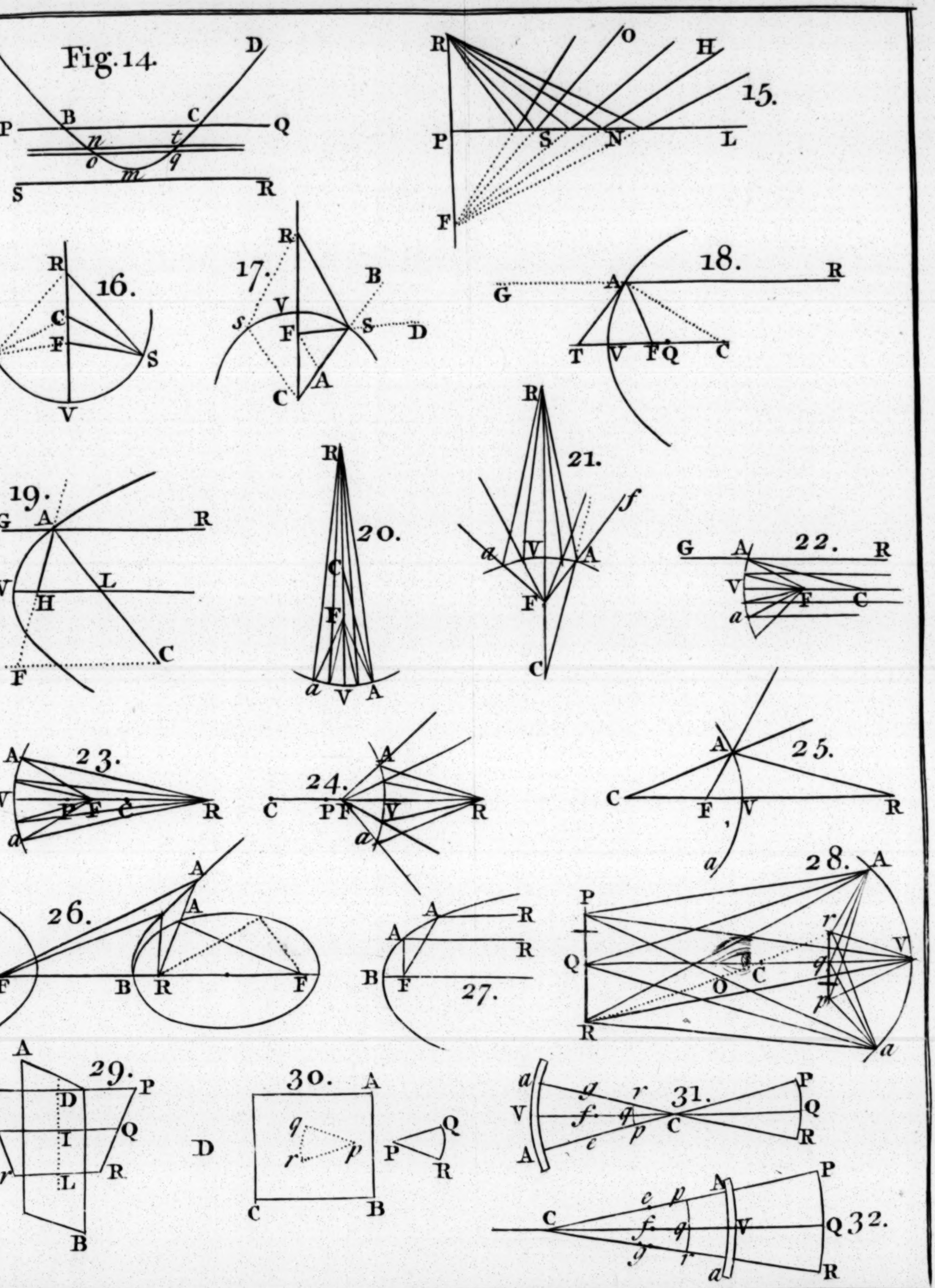


icks.

Pl.II. pa.66.

Fig. 33. 34.

$\frac{ep}{eO} = \frac{op}{OP}$. But if qo be alſo a right line, the figures pqo and PQO will be ſimilar; whence $\frac{Cp}{CP}$ would be $= \frac{op}{OP}$. But $\frac{op}{OP}$ is leſs than $\frac{Cp}{CP}$; for ſince $ep : eC :: eC : eP$, by diviſion $ep : eC :: eC - ep\,(pC) : eP - eC\,(CP)$, therefore $\frac{ep}{eC} = \frac{Cp}{CP}$. But $\frac{ep}{eC}$ is greater than $\frac{ep}{eO}$; therefore taking their equals, $\frac{Cp}{CP}$ is greater than $\frac{op}{OP}$, or $\frac{op}{OP}$ leſs than $\frac{Cp}{CP}$. Therefore oq (the picture of OQ) will not be a right line, but po will be leſs; and therefore oq will be concave towards C.

On the contrary, if oq (concave towards C) be the object, the right line OQ will be the picture. But then a right line oq being an object, will not have the right line OQ for its picture. For as o approaches e, O will go farther from P. And therefore OQ (the picture of the right line oq) will be convex towards V, or concave the contrary way. So that when the image appears on the concave ſide of A*a*, it is always convex towards A*a*, or concave the contrary way, that is, towards the eye.

Again, (*fig.* 34.) ſince pq is the image of PQ an object; if QO becomes an object, then as O goes further from A, its image will advance from p towards e; and therefore the image of the right line OQ will be more curve than before, being convex towards AV.

On the contrary, if the arch pq be an object, the arch PQ will be its image; and therefore if the line oq be an object, where o approaches nearer A, P (the image) will alſo approach nearer A, and the image will then be more concave towards V, Fig. 33. 34. or towards C. So that in all caſes, a concave reflector makes the image concave.

Cor. 1. *The image of a right line is more curve when ſeen beyond any reflector; than when ſeen nearer to the eye, or on the ſame ſide.*

Cor. 2. *Any plane figure ſtanding directly before a concave reflector, will appear concave; and before a convex one, convex.*

Cor. 3. *The image of a ſhort right line or plane, is nearly a right line or plane figure.*

Cor. 4. *A right line at the center of a concave reflector, will appear a right line.*

For then $\frac{op}{OP} = \frac{Cp}{CP}$.

PROP. XV.

35. 36. *The object and its image, ſubtend equal angles at the vertex of the reflector:* $PVR = pVr$.

Let PR be an object, pr its image. Thro' the center C, draw QCV perp. to PR. Draw VP, VR, Vp, Vr, and PpA, Rra, paſſing thro' C. Then (Prop. V.) VQ : Vq : : QC : qC : : (ſimilar triangles) PQ : pq. And ſince the angles at Q and q are right, the angle $PVQ = pVq$. In like manner $RVQ = rVq$; whence $PVR = pVr$.

Cor. *Hence the object and its image are equal to one another, where they meet together.*

Fig. 35. 36.

PROP. XVI.

The linear magnitude or diameter of the object PR, *is to the magnitude of the image pr; as the diſtance of the object from the vertex of the glaſs* VQ, *to the diſtance of the image from it* V*q*.

For (laſt Prop.) the angles PVR and *p*V*r* being equal, and the triangles PVR and *p*V*r* ſimilar, we have $PR : pr :: VQ : Vq$.

Cor. 1. *The diameters or linear magnitudes of the object and image; are as their diſtances from the center of the reflector.* $PR : pr :: CQ : Cq$.

For $PR : pr :: VQ : Vq ::$ (Prop. V.) $CQ : Cq$.

Cor. 2. *If f be the principal focus; then the diameter or linear magnitude of the object, to that of the image; is as the principal focal diſtance, to the diſtance of the image from the principal focus; or as the diſtance of the object from the principal focus, to the principal focal diſtance.*

For we had $PR : pr :: VQ : Vq :: CQ : Cq :: VQ - CQ$ (or $CQ - VQ$) $: Vq - Cq$ (or $Cq - Vq$) $:: CV : 2fq :: fV : fq :: fQ : fV$.

Cor. 3. *Hence alſo it follows, that a convex reflector leſſens an object.*

Cor. 4. *Alſo a concave reflector leſſens an object, when it is farther from the glaſs than the radius of the reflector.*

Cor. 5. *But a concave reflector magnifies an object, when it is nearer to it, than the radius of the reflector; and then only.*

Cor. 6. *The image pr is always terminated at p, by the extreme ray* PA, *which paſſes thro' the center*

Fig. 35. 36. C *of the reflector. And if the object be at an immenſe diſtance, it is terminated by the radius* CA, *drawn parallel to the extreme ray* PV, *at the vertex.*

For when the object is infinitely diſtant, PV and PCA are parallel.

SCHOLIUM.

If it be required to find the place of an object, that its diameter ſhall be to the diameter of the image, in the given ratio of m to 1. It is eaſily done by the VIIIth Prop. For ſince $d : f :: m : 1$, therefore $\frac{d}{m} = f = \frac{dr}{2d+r}$, whence $2d+r = mr$, and $2d = mr - r$, therefore $d = \frac{m-1}{2}r$, for a convex ſpeculum; in which caſe m is always greater than 1. But if m be leſs than 1, then d comes out negative; that is, the object muſt be always on the concave ſide, to have the image magnified. In like manner, any two, d, r, or m being given, the third may be found.

PROP. XVII.

As the diſtance of the object from the principal focus of a reflector, is to its diſtance from the reflector; ſo the principal focal diſtance, to the diſtance of the image from the reflector. $fQ : VQ :: fC : Vq$.

For f being the principal focus, we have (by Prop. XVI.) $VQ : Vq :: PR : pr$. And (by Cor. 2. Prop. XVI.) $PR : pr :: fQ : fV$; therefore $fQ : fV$ or $fC :: VQ : Vq$.

Cor. *If the object and image be equidiſtant from the reflector; they will coincide either in the center or vertex of the reflector, and become equal.*

For if $VQ = Vq$, then $fQ = fC$ or fV, and Q falls upon C, or elſe upon V; and then $pr = PR$.

PROP.

Fig. 35. 36.

PROP. XVIII.

In a convex ſpeculum, the image always appears erect and on the other ſide of the glaſs, as in a plain ſpeculum. And ſo it does in a concave, when the object is between the glaſs and the principal focus. But in a concave ſpeculum, when the object is further from it than the principal focus, the image is on the ſame ſide of the glaſs, and inverted in every reſpect.

For *fig.* 36, let RP be placed where it will before the convex *a*A, its image *rp* will be on the contrary ſide. And if *rp* be an object, its image RP will be on the contrary ſide, till *rp* comes to *f*, and then RP will be infinitely diſtant. But when it is between *f* and C (*fig.* 35.) as at *q*, the image RP is on the ſame ſide. Or if RP be the object, then the image *rp* is on the ſame ſide, but inverted; becauſe the object and image are on different ſides of the center, where the rays croſs. And if the object PR be moved to an infinite diſtance, the image *pr* will be moved to *f*, ſtill on the ſame ſide of the glaſs.

Cor. 1. *If the center is between the object and image, they either both approach to it, or both fly from it. And if the ſpeculum be between them, they either both approach the ſpeculum, or both fly from it.*

For (*fig.* 35.) when C is between Q and *q*, the greater CQ is, the greater is C*q*. And (*fig.* 36.) when V is between them, the greater VQ is, the greater is V*q*.

Cor. 2. *As an object moves from the glaſs on the convex ſide to an infinite diſtance, its image moves from the glaſs to the principal focus. And as the object moves from an infinite diſtance on the concave ſide, to the center, the image moves from the principal focus*

Fig. *to the center alſo, where they meet. And further, if*
35. *the object moves from the center to the principal focus;*
36. *the image moves from the center, the contrary way, to an infinite diſtance. And as the object moves from the principal focus to the vertex, the image moves from an infinite diſtance on the convex ſide to the vertex alſo, where they meet again.*

For (*fig.* 36.) whilſt the object moves thro' VQ, the image moves thro' V*q* to *f*. And (*fig.* 35.) whilſt the object moves thro' QC, the image moves thro' *f*C. Alſo whilſt the object moves thro' C*f*, the image moves thro' CQ to an infinite diſtance; and (*fig.* 36.) whilſt the object moves thro' *f*V, the image moves from an infinite diſtance thro' QV.

Cor. 3. *Tho' the object be removed to an infinite diſtance, from a convex glaſs, yet the image will only recede to the diſtance of half the radius, and will there remain.*

Cor. 4. *The image is upright or inverted, according as the image and object are on the ſame or different ſides of the center.*

PROP. XIX.

To ſee the image of an object made by any ſpherical ſpeculum; the eye muſt be placed in the diverging rays, at a proper diſtance for diſtinct viſion, and facing the image.

For to ſee the image, one muſt look at it, as one would do at any viſible object. And therefore in order to be ſeen, the rays muſt diverge from it to the eye placed at a proper diſtance. Thus to ſee the image *pr* (*fig.* 35.) reflected from the concave ſpeculum A*a*, the eye muſt be placed ſomewhere in the line *q*CQ. But if PR was the image (of the object *pr*), the eye muſt be placed beyond Q. Again (*fig.* 36.) to ſee the image *rq*, reflected by the convex ſpeculum A*a*, the eye muſt be placed without the glaſs in the line VQ. But if *rp* is an object, and RP its image, then the eye muſt be placed in the line VC, on the concave ſide. The focus Q being only a virtual focus, to which the rays only tend to, but never reach. Fig. 35. 36.

Cor. 1. *If the eye be placed near the ſpeculum, in the converging rays, before they reach the image; it will perceive the image (of the object) beyond the glaſs, at the ſame diſtance nearly as the object is before it; and of the ſame magnitude.*

For the eye being near the glaſs, that ſmall part of it which reflects rays to the eye, may be taken for a plane, and therefore the image will appear, as in a plane ſpeculum, at the diſtance of the object behind it. But the rays that come to the eye placed beyond the image, do not fall upon the eye in the ſame manner; for they all diverge from points very near the eye.

Cor. 2. *If an image be looked at, with both eyes placed very near it, and in the diverging rays; (or elſe it cannot be ſeen at all); it will appear double.*

For the axes of the eyes cannot both be directed to an object extremely near; and the caſe is the ſame in all objects near hand, as well as this image.

Cor. 3. *Tho' an eye cannot ſee an image in the air, except it be placed in the diverging rays; yet if that image be received on a white paper, it may be ſeen in any poſition of the eye.*

For the rays reflected from the glaſs, to the image and beyond it, flow but in that one direction. But when the image is received on a white paper, the paper reflects them in all directions, wherever the eye is placed.

Cor. 4. *If the eye be moved whilſt it views the image, the image will appear to be moved.*

For

Fig. For rays from different points of the ſpherical
35. reflector, will come ſucceſſively to the eye.
36.

PROP. XX.

35. *If an object rp be placed in the principal focus of a concave reflector; its apparent magnitude to the eye, at any place whatever as* O, *will be invariably the ſame; and equal to the apparent magnitude to the naked eye, when ſeen from* C *the center of the ſpeculum.*

Here RP the image will be at an infinite diſtance. And (by Cor. 1 and 3, Prop. VI.) all the rays of the pencil *r*BV will be reflected parallel to one another, and to the axis *a*C paſſing thro' the center of the ſpeculum; and therefore the angle VOB, which is the apparent magnitude of the image to the eye at O, is equal to the angle VC*a* or VC*r*, which is the apparent magnitude of *rq* to an eye at C. And this is ſo, wherever O is, or the place of the eye.

Cor. 1. *Hence the apparent magnitude of a body placed in the principal focus, will always continue the ſame, however the eye is moved backward or forward from the reflector.*

Cor. 2. *The nearer the eye is to the ſpeculum, the more of the object appears; and the further off, the leſs.*

For rays parallel to C*r* or BO, that lye beyond O (in reſpect to C), cut the reflector in points beyond B, and at laſt will miſs the reflector, and then the point *r* cannot be ſeen there.

Cor. *If the object be nearer than the principal focus, its apparent magnitude grows leſs in going from the ſpeculum. If it be further off, it encreaſes.*

For if the object be nearer, the image is beyond the ſpeculum, and then OB, C*r* (converging to a point beyond the ſpeculum), makes the angle BOC leſs than *r*C*q*. But if the object is further off, than the principal focus, the image is on the ſame ſide; and then the rays converging to a point beyond O, makes the angle BOV greater than *r*C*q*. Fig. 35.

SCHOLIUM.

Some other phœnomena of concave reflectors, are theſe that follow.

1. The apparent magnitude of an object will be invariable, wherever it is placed, when the eye is at the principal focus.

2. When the eye is fixt at a leſs diſtance than the principal focus; as the object is moved from the ſpeculum, the apparent magnitude decreaſes.

3. When the eye is fixt at a greater diſtance; the apparent magnitude of an object in going from the ſpeculum increaſes, till it comes to the conjugate focus (of the place of the eye), and then it is infinite, and all confuſion. And afterwards it diminiſhes again, and is inverted.

4. A man's face in a reflecting concave, in going from it, decreaſes to the principal focus, and then increaſes.

PROP. XXI.

The brightneſs of the image pr formed by the reflecting concave AB, *from the luminous object* PR, *is as* $\frac{AB^2}{Vq^2}$, *or as the area of the lens directly, and the ſquare of the diſtance of the image reciprocally.* 35.

For if the brightneſs of the object be given, the quantity of light reflected from the ſpeculum to the image, will be as the magnitude of the object and of the reflector directly, and the ſquare of the diſtance

Fig. 35. diſtance reciprocally. Whence the light falling on the image is as $\frac{PR^2 \times AB^2}{QV^2}$; alſo the denſity of the light, or the brightneſs of the image is as the quantity of light directly, and the area of the image reciprocally, or as $\frac{PR^2 \times AB^2}{QV^2 \times pr^2}$. But ſince PR, *pr* are parallel, and (Prop. XV.) the angle PVR $=$ *p*V*r*, &c. PR : *pr* : : QV : *q*V, and $\frac{PR^2}{QV^2} = \frac{pr^2}{qV^2}$. Therefore the brightneſs of the image is as $\frac{pr^2 \times AB^2}{qV^2 \times pr^2}$ or $\frac{AB^2}{qV^2}$.

Cor. 1. *If the brightneſs of the object be increaſed in any ratio, which call* B; *the brightneſs of the image will then be as* $\frac{AB^2}{qV^2}$ B.

Cor. 2. *In very remote objects, if f be the principal focal diſtance; then the brightneſs of the image will be as* $\frac{AB^2}{ff}$ B.

For then V*q* becomes equal to *f*.

Cor. 3. *The brightneſs of the image increaſes as the area of the glaſs increaſes; but its diſtinctneſs decreaſes upon that account.*

For the rays further from the vertex are reflected to different points of the axis, as appears from the different proportions in Prop. III. and V.

PROP.

PROP. XXII.

If a concave ſpeculum Aa *be held directly to the rays of the ſun; and any combuſtible body be held in the principal focus f, it will be fired by the heat thereof.* 35.

For all the rays of the ſun are reflected into the focus, forming there, by reflexion, the image of the ſun. Which image muſt be of an intenſe heat, becauſe all the rays that fall upon the ſpeculum, are brought thither. And the broader the ſpeculum is, the more rays are contained in that ſmall ſpace; and may be ſo increaſed by the breadth of the glaſs, as to burn or melt all ſorts of bodies placed there; as is plain from numerous experiments.

Cor. 1. *The heat generated in the focus of the glaſs, is to the ſun's direct heat; as the area of the glaſs, is to the area of the image in the focus.*

For the rays that fell upon the glaſs are all brought into that ſpace, where their denſity is reciprocally as the areas they are contained in.

Cor. 2. *The degrees of heat in different reflectors, are as the ſquares of the diameters directly, and the ſquares of the focal diſtances reciprocally.*

For the denſity of the rays in each, are in that proportion.

SCHOLIUM.

Upon account of the rays meeting in that point, and burning bodies therein, it is called the *focus* or *burning point*; and the reflector itſelf a *burning concave.* For a reflector to burn the moſt intenſly, ſee Schol. Prob. V. B. IV.

PROP.

Fig.

PROP. XXIII.

37. *If parallel rays* RA, CV, *are reflected from a concave (or convex) ſpherical ſurface* A*a*, *of a ſmall hight* VP. *Let* F *be the focus of the extreme ray* AF, Q *the principal focus. Then the aberration of the rays, in the length of the axis* VC, *is equal to half the hight of the ſegment*; $QF = \frac{1}{2}VP$, *nearly.*

For draw the tangent AT. Then by the nature of the circle CP : CV : : CV : CT. And dividing, CP : CV : : CV — CP (VP) : CT — CV (TV). But VP being very ſmall, CP is = CV nearly, therefore VP = VT. But (Prop. IV.) TF = FC, or $CF = \frac{1}{2}CT$. And (Prop. VI.) VQ = QC, or $CQ = \frac{1}{2}CV$. Therefore CF — CQ or $FQ = \frac{CT - CV}{2} = \frac{TV}{2} = \frac{VP}{2}$. But (Cor. 4. Prop. IV.) QF is the ſpace on which all the rays fall that come from the whole ſurface A*a*, that is, QF is the aberration of the rays in length; therefore $\frac{1}{2}VP$ is equal to that aberration.

Cor. 1. *The aberration* $QF = \frac{1}{2}TV$ *exactly.*

Cor. 2. *The aberration* $QF = \frac{AV^2}{4VC}$, *nearly.*

For $VP = \frac{VA^2}{2VC}$, and $\frac{1}{2}VP = \frac{VA^2}{4VC}$.

Cor. 3. *The lateral aberration at the principal focus is* $= \frac{VA^3}{2VC^2}$, *nearly.*

For the triangles APF, DQF are ſimilar, whence PF : PA : : FQ : QD; that is, $\frac{1}{2}CV : AV : : \frac{VA^2}{4CV} : QD = \frac{VA^3}{2VC^2}$, nearly; becauſe AV is very ſmall.

Cor. 4. *The radius of the ſphere being given; the aberrations in length, are as the ſquares of the diameters; and the lateral aberrations, as the cubes of the diameters of the reflectors.* Fig. 37.

PROP. XXIV.

If parallel rays RA, SG, *are reflected from a concave (or convex) ſpherical ſurface* A*a*; *the diameter of the leaſt circle of aberration of the rays, is* $\frac{1}{4}$ *the aberration made at the principal focus.* BI $= \frac{1}{4}$QD. 38.

Let any reflected ray GB cut the axis in L, and the extreme ray AF in B. Draw BI perp. to VQ, Q being the principal focus. Then ſuppoſing the ray AFD to be fixed; as the point of incidence G moves from the vertex V, the perpendicular BI, will firſt increaſe, becauſe the angle VLG continually increaſes; and afterwards it will decreaſe, becauſe the line FL continually decreaſes. And when BI is the greateſt, it is plain, all the rays, on the ſame ſide, will paſs thro' it; and then the circular ſection of all the rays, in that place I, will be the leaſt ſection poſſible. To find the point I, or the radius BI.

By ſimilar triangles, LI : BI : : LP or FP : PH,
and BI : FI : : AP : PF.
and ex equò LI : FI : : AP : PH.
and compounding LI + FI (FL) : FI : : AP + PH (AH) : PH.
Again QF : QL : : AP^2 : PH^2 (by Cor. 4. Prop. 23.)
and by diviſion, QF : QF — QL (FL) : : AP^2 : AP^2 — PH^2 (AH × H*a*).
but we had FL : FI : : AH PH
therefore QF : FI : : AP^2×AH : PH×AH×H*a*.
or QF : FI : : AP^2 : PH×H*a*.
but we had FI : BI : : FP : AP
therefore QF : BI : : AP^2×FP : AP×PH×H*a*
or QF : BI : : AP × FP : PH×H*a*.

But

Fig. 38. But QF, AP, FP are given; therefore BI is the greateſt when $PH \times Ha$ is the greateſt, that is, when $PH = Ha, = \frac{1}{2}AP$, and then $PH \times Ha = \frac{1}{4}AP^2$. And ſince $AP^2 : PH \times Ha \left(= \frac{1}{4}AP^2\right) :: QF : FI \left(= \frac{1}{4}FQ\right)$, and conſequently $BI = \frac{1}{4}QD$.

Cor. 1. *If* R *be the radius of the reflector*, B = *half its breadth; then the diameter of the leaſt circle of aberrations cauſed by the figure of the ſpherical reflector, will be* $\frac{B^3}{4RR}$.

For (Cor. 3. Prop. 23.) $QD = \frac{VA^3}{2RR} = \frac{B^3}{2RR}$, and 2BI, or $\frac{1}{2}QD = \frac{B^3}{4RR}$.

Cor. 2. *The diſtance* FI *of the leaſt circle of aberrations from* F *the focus of the extreme ray, is* $\frac{1}{4}$ FQ, *or* $\frac{1}{4}$ *the diſtance from the principal focus.*

Cor. 3. *If this circle of aberrations be ſet in any other place, all the rays will not paſs thro' it.*

Cor. 4. *And therefore the circular ſection cutting all the rays, will be greater at any other place, beſides* I.

Cor. 5. *The area of the circle of aberrations, is to the area of the glaſs; as* VA^4 *to* $64R^4$. R *being the radius of concavity.*

For the area of the glaſs, is to the area of the circle; as VA^2 to BI^2, or as VA^2 to $\frac{B^6}{64R^4}$; or as $64R^4$ to B^4; B being = VA.

PROP.

PROP. XXV.

Fig. 39. *Let* R *be the radiant point,* F *the focus of the rays falling at* A, *f the focus of thoſe falling at* V. C *the center of the reflector. Then will* $FC = \frac{CR \times CA^2}{AR^2 - CR^2}$, *and* $fC = \frac{CR \times CA}{VC + 2CR}$. *And the aberration in length is* $= FC - fC$.

Let radius CA or CV $= r$, AR $= a$, CR $= d$, AF $= z$, FC $= y$, Then (Geom. II. 25.) $a : d :: z : y$, and $dz = ay$, and $z = \frac{ay}{d}$. Alſo (Geom. II. 26.) $rr + dy = az$, and $z = \frac{rr + dy}{a} = \frac{ay}{d}$. Whence $drr + ddy = aay$, and $\overline{aa - ad} \times y = drr$, and $y = \frac{drr}{aa - dd}$.

Again (Prop. V.) VR $(r + d)$: Vf $(r - fC)$ :: CR (d) : Cf. And by compoſition, $r + 2d : d :: r : Cf = \frac{dr}{r + 2d}$. And the aberration $= \frac{drr}{aa - dd} - \frac{dr}{r + 2d}$.

Cor. 1. *On the contrary, if* F *be the radiant point,* R *the focus of rays falling on* A, *r the focus of rays falling on* V; C *the center of the reflector. Then will* $CR = \frac{CF \times CA^2}{AF^2 - FC^2}$, *and* $Cr = \frac{CF \times CA}{VC - 2CF}$. *And the aberration* $Rr = Cr - CR$.

For from the equations $dz = ay$, and $rr + dy = az$, we have $a = \frac{dz}{y} = \frac{rr + dy}{z}$, and $dzz = rry + dyy$. Whence d or CR $= \frac{rry}{zz - yy}$.

Fig. 39. Again (Prop. V.) FV $(r-y)$: FC (y) : : Vr $(r+Cr)$: Cr. By diviſion, $r-2y : y :: r : Cr = \frac{ry}{r-2y}$. Therefore $Cr - CR = \frac{ry}{r-2y} - \frac{rry}{zz-yy} = Rr$.

Cor. 2. *If* R *and* F *are conjugate foci, for rays falling at* A; *then* R *and* f, *alſo* r *and* F, *will be conjugate foci for rays near the axis at* V. *And* f *and* r, *will be further from the vertex than* F *and* R.

That f and r are further from the vertex, will appear thus. AC + CR is greater than AR; that is, VC + CR or $r+d$ is greater than a, and $rr + 2rd + dd$ greater than aa, and $rr + 2rd$ greater than $aa - dd$. Whence $\frac{drr}{aa-dd}$ is greater than $\frac{drr}{rr+2rd}$ or $\frac{dr}{r+2d}$; that is, FC is greater than fC.

Again, AF + FC is greater than AC or VC, or VF + FC, and AF greater than VF; that is, z is greater than $r-y$, and zz greater than $rr - 2ry + yy$, and $zz - yy$ greater than $rr - 2ry$. Therefore $\frac{rry}{rr-2ry}$ or $\frac{ry}{r-2y}$ is greater than $\frac{rry}{zz-yy}$; that is, Cr is greater than CR.

Cor. 3. *Therefore if* R, f *be conjugate foci to the vertex* V; R *and* F, *will be conjugate foci to the point* A. *And if* F *and* r *be conjugate foci to the vertex* V; *then* F *and* R *will be conjugate foci to the point* A.

Cor. 4. *Let* F, r, *be the conjugate foci to the vertex* V; *then the aberration* $Rr = \frac{VP \times rC \times 2CF}{CV^2 - 2CF \times CP}$, *for rays falling at* A.

For let $x = \mathrm{VP}$, $v = r - 2y$; and $rr = yy +$ Fig. $zz + 2y \times \mathrm{FP}$; and $zz = rr - yy - 2y \times \overline{r - y - x}$, 39. and $zz - yy = rr - 2ry + 2yx. = rv + 2yx$.

Then $\mathrm{R}r = \frac{ry}{r - 2y} - \frac{rry}{zz - yy} = \frac{ry}{v} - \frac{rry}{rv + 2yx}$

$$= \frac{ry}{v} - \frac{ry}{v + \frac{2yx}{r}} = \frac{vry + 2yyx - vry}{v \times \overline{v + \frac{2yx}{r}}} =$$

$$\frac{2yyx}{v \times \overline{v + \frac{2yx}{r}}} = \frac{x \times \frac{y}{v} \times 2y}{v + \frac{2yx}{r}} = \frac{x \times \frac{ry}{v} \times 2y}{rv + 2yx} =$$

$$\frac{x \times \frac{ry}{r - 2y} \times 2y}{rr - 2ry + 2xy} = \frac{\mathrm{VP} \times \mathrm{C}r \times 2\mathrm{CF}}{\mathrm{CV}^2 - 2\mathrm{CF} \times \mathrm{CP}}.$$

Cor. 5. *If* T *be the principal focus. And let* TF : TC : : 1 : n; *then the aberration* $\mathrm{R}r =$

$$\frac{\overline{n - 1} \times \mathrm{VP} \times \mathrm{VT}}{\frac{\mathrm{VC}}{n - 1} + \mathrm{VP}}.$$

For by diviſion TF : TC — TF or FC : : 1 : $n - 1$; that is, $\frac{1}{2} r - y : y : : 1 : n - 1 =$

$\frac{y}{\frac{1}{2}r - y} = \frac{2y}{r - 2y}$, and $\frac{r \times \overline{n - 1}}{2} = \frac{ry}{r - 2y}$. And

$$\text{(Cor. 4.) } \mathrm{R}r = \frac{x \times \frac{ry}{r - 2y} \times 2y}{rr - 2ry + 2xy} = \frac{x \times \frac{r \times \overline{n - 1}}{2} \times 2y}{rr - 2ry + 2xy}$$

$$= \frac{\overline{n - 1} \times \frac{1}{2} rx}{\frac{rr - 2ry}{2y} + x} = \frac{\overline{n - 1} \times \frac{1}{2} rx}{\frac{r}{n - 1} + x}; \text{ becauſe } n - 1$$

$$= \frac{2y}{r - 2y}, \text{ and } \frac{r}{n - 1} = \frac{rr - 2ry}{2y}.$$

Fig.

PROP. XXVI.

40. *If* R *be the radiant point,* F *the focus of rays falling on* A, *and reflected by the convex ſpeculum* AV, *whoſe center is* C, *f the focus of rays falling on* V. *Then will the aberration* $Ff = \frac{RC \times CA^2}{RC^2 - RA^2} - \frac{RC \times CA}{2RC - CV}$.

Draw RH parallel to the reflected ray AG, and RO perp. to CA produced. Then $<$ H $=$ FAC $=$ CAS $=$ RAH, and conſequently RH $=$ RA, and AO $=$ OH $= \frac{1}{2}$ AH. Let CA or CV $= r$, CR $= d$, RA or RH $= a$, CF $= y$, AF $= z$, AO $= v$, VP $= x$.

Then in the triangle RCA, (Geom. II. 22.) $dd = aa + rr + 2vr$, and $dd - aa = rr + 2vr = \overline{r + 2v} \times r$. And by the ſimilar triangles CFA, CRH, CR (d) : CF (y) : : CH $(r + 2v)$: CA (r) : : $\overline{r + 2v} \times r : rr : : dd - aa : rr$. Whence $y = \frac{drr}{dd - aa} =$ CF.

Alſo (Prop. V.) VR $(d - r)$: Vf $(r - Cf)$: : CR (d) : Cf. And compounding, $2d - r : d : : r : Cf = \frac{dr}{2d - r}$. And F$f$ $=$ CF $-$ Cf.

Cor. 1. *If* R, *f be the conjugate foci, to the vertex* V; *then for rays falling on* A, *the aberration* $Ff = \frac{Cf \times VP}{\frac{CV^2}{2Cf} - VP}$; *f being the focus of rays falling at* V.

For (Geom. II. 23.) CP $= r - x$, and C being an acute angle, we have in the triangle RAC, $aa = rr + dd - 2d \times \overline{r - x}$, and $dd - aa = 2dr - 2dx$

$2dx - rr$. Therefore $Ff = \frac{drr}{2dr - rr - 2dx} - \frac{dr}{2d - r} = \frac{drr}{2dr - rr - 2dx} - \frac{drr}{2dr - rr} =$ Fig. 40.

$$\frac{2dr - rr - 2dr + rr + 2dx}{\overline{2dr - rr - 2dx} \times \overline{2dr - rr}}\, drr = \frac{2drx}{\overline{2dr - rr - 2dx} \times \overline{2dr - rr}}\, dr = \frac{\frac{dr}{2d - r} \times x}{2dr - rr - 2dx}$$

$$\times\, 2d = \frac{\frac{dr}{2d - r} \times x}{\frac{2d - r}{2d} r - x} = \frac{\frac{dr}{2d - r} x}{\frac{2d - r}{dr} \times \frac{1}{2} rr - x} = \frac{Cf \times x}{\frac{rr}{2Cf} - x}.$$

Cor. 2. *Let* T *be the principal focus, and* CT *or* TV $= t$. *And let* TR : TV : : m : 1. *Then the aberration* $Ff = \frac{\frac{m + 1}{m} tx}{\frac{m}{m + 1} \times \overline{2t - x}}$.

For TR $(d - \frac{1}{2} r)$: TV $(\frac{1}{2} r)$: : m : 1, or $2d - r : r : : m : 1$, and by addition $2d : r : : m + 1 : 1$. Hence $m = \frac{2d - r}{r}$, and $m + 1 = \frac{2d}{r}$, therefore $\frac{m + 1}{m} = \frac{2d}{2d - r}$, and $\frac{m + 1}{2m} r = \frac{dr}{2d - r}$ $= Cf$. Whence $\frac{Cf \times x}{\frac{rr}{2Cf} - x} = \frac{\frac{m + 1}{2m} rx}{\frac{rr}{2} \times \frac{2m}{\overline{m + 1} \,.\, r} - x} =$

$$\frac{\frac{m + 1}{m} \times \frac{1}{2} rx}{\frac{m}{m + 1} \times r - x} = \frac{\frac{m + 1}{m} tx}{\frac{m}{m + 1} \times \overline{2t - x}}.$$

Fig.

PROP. XXVII.

41. *Let* R *be the radiant point*, RA *the incident ray falling upon the ſpherical ſurface* AV, AD *the reflected ray*, C *the center of the reflector*; *draw* CB, CD *perp. to* RA, AD. *And let* T, *t*, *be the middle points of* AB, AD. *Then if* F *be the focus of the reflected ray, it will be as* RT : TA : : TA *or* *t*A : *t*F. *Where* F *muſt lie the ſame way from t*, *as* R *lies from* T.

Let R*a* be another ray infinitely near RA; *a*F*d* its reflected ray; draw C*b*, A*n* perp. to R*a*; and C*d*, A*s* perp. to F*a*. Then the right angled triangles A*as*, and A*an* are equal and ſimilar; for < *na*A = F*a*V = S*a*A, and A*a* is common, therefore A*s* = A*n*. Alſo in the equal right angled triangles ACB, ACD; and alſo *a*C*b*, *a*C*d*; CB = CD, and C*b* = C*d*, therefore B*b* = D*d*. And by the ſimilar triangles RB*b*, RA*n*; and FA*s*, FD*d*; RB : RA : : B*b* or D*d* : A*n* or A*s* : : FD : AF. And by compoſition and diviſion, RA + RB : RA — RB : : AF + FD : AF — FD. And

$$\frac{RA + RB}{2} : \frac{RA - RB}{2} :: \frac{AF + FD}{2} : \frac{AF - FD}{2}.$$

That is, RT : TA : : A*t* : *t*F.

Cor. 1. *This Prop. holds true of any curve* AV, *whoſe radius of curvature at* A, *is* AC.

Cor. 2. RA : RB : : AF : FD.

Cor. 3. T *and t are the foci of rays parallel t* RA, FA.

Cor. 4. *The middle point of* CA *is the focus o rays parallel to* CA.

Cor 5. *If rays fall on the convex ſide, it is only putting* R *on the other ſide of* AV; *and the ſame proportion holds good.* Fig. 41.

SCHOLIUM.

Altho' it appears by Prop. IX, and X. that a conic ſection will reflect all rays iſſuing from one focus, accurately to the other focus; and therefore mathematicians have endeavoured to grind their glaſſes into the form of ſome of the conic ſections, yet they have never ſucceeded in giving them a true figure, nor has any method yet been found out to grind them accurately into that form. And if they could, it would not anſwer their expectation; for neither the conic ſections, nor any other curve, will reflect the rays iſſuing from any point in the collateral parts of an object, accurately to a ſingle point. So that if glaſſes were truly made in the form of any of the conic ſections, they would not perform much better than thoſe of a ſpherical form. For near the vertex of a conic ſection, it nearly coincides with a ſphere of a certain radius, and therefore in a ſmall ſegment, they will nearly reflect alike. And ſpherical glaſſes are eaſily ground into a true form. And therefore conſidering all things, a ſpherical ſurface is the beſt curve, and the moſt commodious for optical uſes of any.

BOOK III.

DIOPTRICS, *or refracted Viſion.*

DEFINITIONS.

DEFIN. I.

Fig. *DIOPTRICS* is that part of Optics, whic treats about refracted light, and every thin that relates to ſeeing thro' different mediums, an the pictures of objects made by refraction throug them.

DEF. II.

43. *A Lens* is a ſolid tranſparent body as AB, gene-rally of glaſs, being of ſuch a ſhape, that the ſide of it are either flat, or the ſegments of ſpherica ſurfaces. Of theſe there are ſeveral ſorts, as, 1. *Plain* on both ſides. 2. *Plano convex*, being plai on one ſide and convex on the other. 3. *Plan concave*, or plane on one ſide and concave on th other. 4. *Double convex*, being convex on bot ſides. 5. *Double concave*, being concave on both ſides. 6. A *Meniſcus*, being convex on one ſide, and concave on the other.

DEF. III.

The *Axis* of a lens, is a right line perpendicu-lar to both the ſurfaces, or paſſing thro' both their centers.

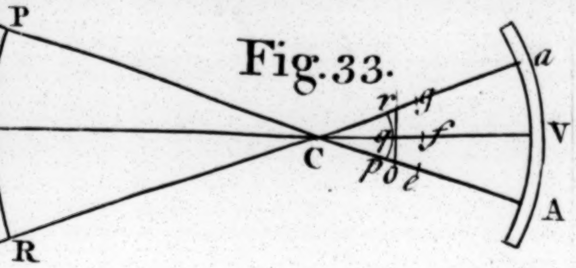
DEF. IV.

The *Vertex* is the point where the axis cuts the ſurface of the lens.

PROP.

Opticks.

Fig. 33.



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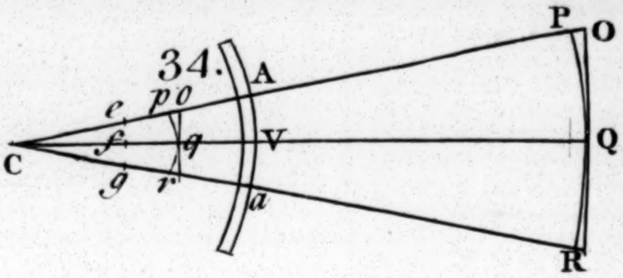

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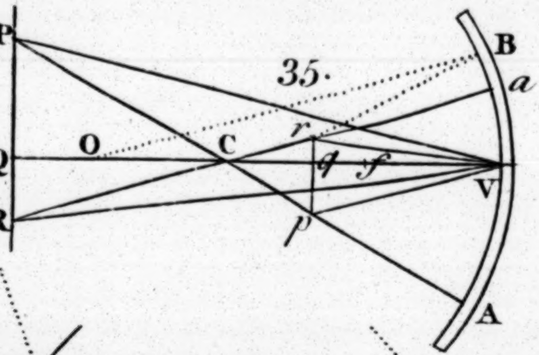

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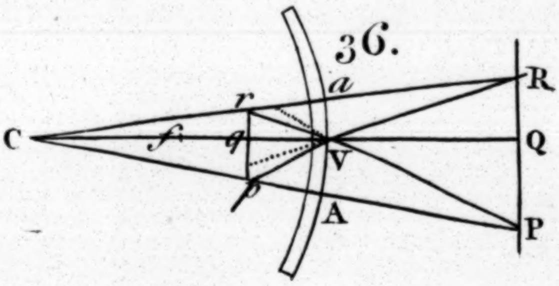

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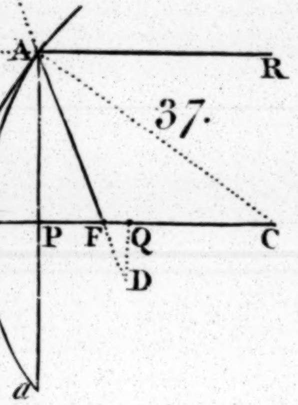

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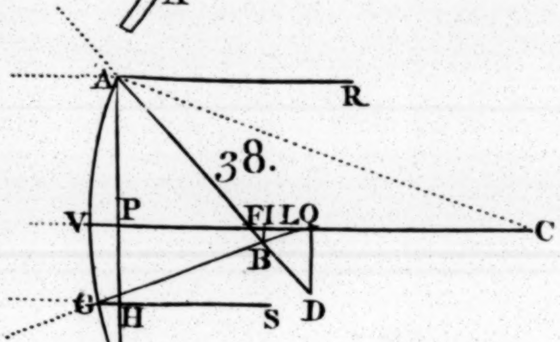

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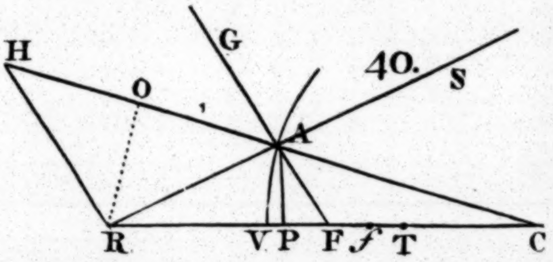

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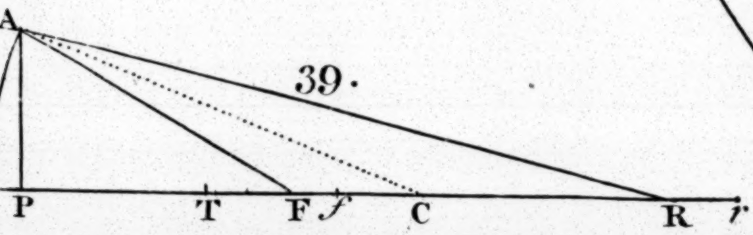

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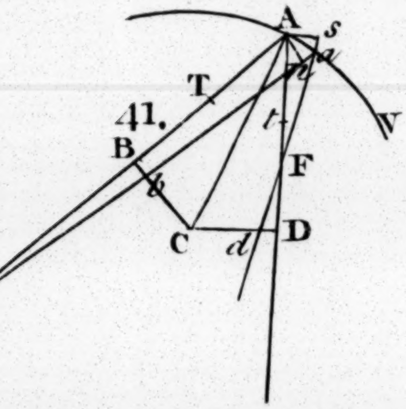

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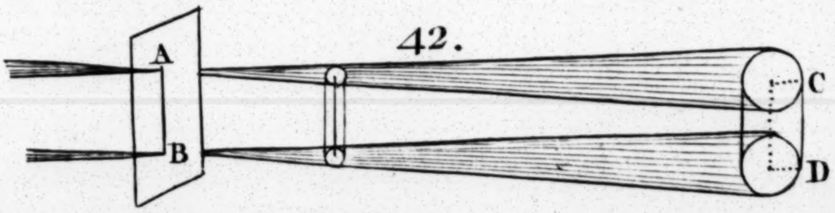

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Pl. III. *pa. 88.*

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Fig.

PROP. I.

If a ray of light AC, *paſs into a refracting medium*; *the ſine of incidence* ACD, *is to the ſine of refraction* ECF, *in a conſtant ratio.* 44.

Let the ray BC falling moſt obliquely on the refracting medium MS, be refracted in the line CN. And the ray ACI be refracted into the line CE. Draw AP perp. to MS, and make, CH = CP, and CL = CM. And draw HIE and LN perp. to CS. And let the equal lines BC, AC, repreſent the equal motions of the rays B and A. Then ſuppoſing BC parallel or nearly coinciding with the refracting plane; let the motion AC be reſolved into the two motions, AP, PC; AP being perpendicular, and PC parallel to the refracting ſurface.

Then (by Prop. XVI. B. I.) bodies act upon light by the forces of attraction and repulſion; and theſe forces are directed in lines perp. to the ſurface of the refracting body MS. Therefore the motions of the rays BC, PC which are parallel to MS, ſuffer no alteration by the refraction of the medium. Therefore in the very ſame time that the rays A and B move to C, they would move to I and L without refraction, where CL = CB = CA = CI. But by the refraction of the medium, that from B goes to N, and that from A goes to E, in the perpendiculars LN, HE, becauſe the parallel motion is the ſame as before. But (by Cor. 3. Prop. XX. Mechan.) the velocities of the rays A, B, which were equal before refraction, will alſo be equal after; and therefore CN = CE.

Now in the right angled triangle CLN, we have S.CLN (MCD) : : S.CNL (NCF) : : CN : CL : : CE : CI : : (in the triangle CEI) S. CIH (ACD) :

Fig. (ACD) : S.CEH (ECF). That is, S.ACD the
44. angle of incidence : S.ECF the angle of refraction : : rad : S.NCF, or in a given ratio.

Cor 1. *If the refracted ray* EC *be turned directly back to the point of incidence; it will be refracted into the line* CA, *before deſcribed by the incident ray.*

For the forces that before accelerated its motion, will now equally retard it.

Cor. 2. *The angles of incidence and refraction lye both in one plane, which is perp. to the refracting ſurface at the point of incidence.*

For the refraction being made in lines perp. to the refracting ſurface, there is no force to cauſe the rays to deviate from that plane.

Cor. 3. *Heterogeneal rays of light have different degrees of refraction; but in any one conſidered by itſelf, the ſine of incidence is in a given ratio the ſine of refraction.*

Cor. 4. *If* AC *be the incident ray, and* CE *the refracted ray; and if* AC *produced cut the perpendicular* HE *in* I. *Then* CE : CI : : *as ſine of the angle of incidence : to the ſine of the angle of refraction.*

Cor. 5. *The velocity of light in the firſt medium* ADC, *is to its velocity in the ſecond medium* CFN; *as the ſine of the angle of refraction, to the ſine of the angle of incidence.*

For it is as AC *or* CI *to* CE.

Cor. 6. *If parallel rays paſs out of one medium into another, they will alſo be parallel after refraction: the refracting ſurface being a plane.*

For the rays being parallel, they are equally refracted.

PROP. II.

If heterogeneal light be refracted out of glaſs into air; the ſine of incidence, is to the ſine of refraction of the leaſt, and moſt refrangible rays; as 50 *to* 77 *and* 78.

This is proved by experiment thus; take a glaſs prism BCD, with a very acute angle C, and place it ſo, that a ray of the ſun AF coming thro' a ſmall hole in the window ſhut, near the priſm, may fall perpendicularly upon the ſide DC, and be refracted by the oppoſite ſide at Y, the moſt refrangible rays to P, and the leaſt refrangible to T. Then if EYG be perpendicular to the ſide CB, by meaſuring the angles AYE, TYG and PYG, the ſines of theſe angles will become known. 45.

For example, if the angle of the priſm BCD be 31° 36′; then ſince the triangles CFY and CYE are both of them right angled, the angle BCD = FYE the angle of incidence at Y = 31 36, whoſe ſine is .524. And the angles TYG and PYG will be 53 36, and 54 44½; whoſe ſines are .805 and .8165. But .524 and .805 and .8165 are nearly to one another as 50, 77 and 78.

Here the angle of incidence on the plane BC is always equal to the angle of the priſm BCD; but it may be difficult to meaſure the angles TYG and PYG truly, for want of drawing the perpendicular YG exactly. Therefore to avoid any error, take the altitude of the ſun, and at the ſame time mark the points P, T. Then to the altitude of the ſun, add the angle of the priſm, which was ſhewn to be equal to FYE; the ſun gives the elevation of the perpendicular YG above the horizon. Then if you take alſo the elevation of TY, PY above the horizon; theſe taken from the elevation of GY, will give the angles TYG, PYG.

Cor.

Fig. 45. Cor. 1. *The angle* PYT, *which the coloured image ſubtends, is about* $1^\circ\ 8'\frac{1}{2}$, *or* $1^\circ\ 9'$, *ſubtracting the ſun's diameter.*

Cor. 2. *In the coloured image* PT, *the lengths of the ſeveral colours, red, orange, yellow, green, blue, purple, violet, are found to be* $\frac{1}{16}$, $\frac{3}{80}$, $\frac{1}{15}$, $\frac{1}{12}$, $\frac{1}{12}$, $\frac{1}{18}$, $\frac{1}{9}$; *it is eaſy to divide the angle* PYT *in that proportion; and then there will be had the ſines of refraction at the confines of all the colours; that is,* 77, $77\frac{1}{8}$, $77\frac{1}{5}$, $77\frac{1}{3}$, $77\frac{1}{2}$, $77\frac{2}{3}$, $77\frac{7}{9}$, 78.

For the image is divided after the manner of a muſical cord, as Sir J. Newton has found. And in ſmall arches, the differences of the ſines are nearly as the differences of the arches.

Cor. 3. *If light paſs out of glaſs into air; the ſine of incidence, is to the ſine of refraction of the mean rays; as* 11 *to* 17, *or as* 20 *to* 31; *which is a greater refraction than as* 2 *to* 3.

For the mean refrangible rays are thoſe at the confines of the blue and green, or nearer the green, whoſe refraction is $77\frac{1}{4}$ or $77\frac{1}{2}$.

SCHOLIUM.

By a like experiment, the ſine of incidence and refraction of the leaſt refrangible rays out of water into air is as 3 to 4, or as 81 to 108; and of the moſt refrangible, as 81 to 109.

PROP.

Fig.

PROP. III.

If A, B, C, *be three refracting mediums, contained between parallel ſurfaces; and if a ray of light* DE *paſſing thro' them be refracted at* E *and* F, *and paſs thro' the third in direction* FG. *It will ſuffer the ſame refraction by paſſing thro'* A, B, C, *as if it went immediately out of* A *into* C. 46.

For let the medium C approach the medium A in direction FE, and the medium B grow thinner and thinner, till F fall upon E, and the medium B be vaniſhed; all this makes no difference in the poſition of the line FG. And therefore the refraction into the medium C will be the ſame whether the ray paſs thro' B, or goes immediately from A to C, without coming into B at all. And the ſame likewiſe appears from experiments.

Cor. 1. *Hence if there be never ſo many refracting mediums contained between parallel ſurfaces; the whole refraction of a ray is the ſame at laſt, in paſſing thro' all of them; as it would be in paſſing immediately out of the firſt into the laſt.*

Cor. 2. *If ſeveral mediums* A, B, C, *terminated by parallel planes, be encompaſſed on all ſides by ſome other medium; any ray* RD *moving out of this other medium, and paſſing thro' all the reſt, will emerge parallel to itſelf, at coming into this other medium again; that is,* GS *will be parallel to* RD.

For the refraction is the ſame as if all the mediums A, B, C were away; and GS has the ſame direction as RD.

PROP.

Fig.

PROP. IV.

47. *If* A, B, C *be three mediums, and the ſines of incidence and refraction out of* A *into* B, *be a and b; and out of* A *into* C, *be a and c; then thoſe out of* B *into* C, *will be b and c.*

Let the mediums B and C be encompaſſed by the medium A; and let RD be a ray refracted at D, F, and G, and proceeding to K. Then (Prop. III. Cor. 2.) GK will be parallel to RD. At D, F, G, let fall the perpendiculars LDO, NFP, IGH. Then by reaſon of the parallels GK, RD, and GH, DL, the angle HGK = LDR. Whence $a =$ S.RDL, $b =$ *s*.ODF = S.DFN = S. incidence of the ray DF on the medium C. Alſo $a =$ S.HGK, and conſequently $c =$ S.FGI = S.GFP = S. refraction of the ray into the medium C. Therefore the ſines of incidence and refraction out of B into C, are *b* and *c*.

Example. Let A be air, B water, and C glaſs: And let the ſines of incidence and refraction out of air into water be as 4 to 3; and out of air into glaſs as 17 to 11, or as 4 to $\frac{4 \times 11}{17}$; then $a = 4$, $b = 3$, $c = \frac{4 \times 11}{17}$. Therefore the ſine of incidence and refraction out of water into glaſs will be 3 and $\frac{4 \times 11}{17}$, or as 3×17 to 4×11, or as 51 to 44.

Cor. *If* B, C, D, *be three mediums; and the ſine of incidence and refraction out of* B *into* C, *be as b to c; and out of* C *into* D, *as r to s. Then thoſe out of* B *into* D *will be as br to cs.*

For by this Prop. the ſines of incidence and refraction out of C into B are as c and b; and out of C into D, as r to s, or as c to $\frac{cs}{r}$. Therefore thoſe out of B into D, will be as b to $\frac{cs}{r}$, or as br to cs. Fig. 47.

Examp. If the ſines of incidence and refraction out of glaſs into air, be as 20 to 31; and out of air into water, as 4 to 3. Then the ſines of incidence and refraction out of glaſs into water, will be as 20×4 to 31×3, or as 80 to 93.

PROP. V.

The forces of bodies to refract light, are nearly as their denſities; excepting that unctious and ſulphureous bodies refract more than others of the ſame denſity.

In uniformly accelerated motion, (by Prop. V. Cor. 2. Mechan.) if the body is given, the velocity is as the force and time; and the ſquare of the velocity, as force × time × velocity; that is, (by Cor. Prop. VI. ib.) as the force × ſpace deſcribed, and that is (when the ſpace is given) as the force. Let AF be the refracting ſurface, and let 49.
it be divided into an infinite number of equal parts terminated by parallel planes paſſing thro' A, B, C, D, &c. Put $a =$ AB or BC, &c. f, g, h, &c. for the forces acting in AB, BC, CD, &c. which may be eſteemed uniform. Then the velocities acquired in falling from reſt, thro' AB, BC, CD, will be as $\sqrt{f}$, $\sqrt{g}$, $\sqrt{h}$; and their ſquares as f, g, h. But (by Cor. 3. Prop. XX. Mechan.) the ſquare of the velocity at B being f, the ſquare of the velocity at C is $=$ ſquare of the velocity at B $+$ the ſquare of the velocity in paſſing thro' BC from reſt $= f + g$. And the ſquare of the velocity at D $=$ ſquare of the velocity at C +

Fig. C + the ſquare of the velocity in paſſing thr
49. CD $= f + g + h$. And ſo on, therefore the ve
locity at F $= \sqrt{\overline{f + g + h}}$ &c. = ſquare root
the force.

48. Let AB be the refracting ſurface, IC the inci
dent ray, infinitely near AC, and CR the refracte
ray. Let CB = CA, and BR perp. to CB. The
CB (= CI = 1) is the parallel motion of the ray
and BR the perpendicular motion, which is gene
rated by the refraction. And therefore BR is a
the ſquare root of the refracting force, as was ſhew
before; and the refracting force is as the denſity
therefore the ſquare of BR is as the denſity of th
medium.

Let the ſines of incidence and refraction be a
m to n. Then we have $m : n :: CR : CB =$
and CR $= \frac{m}{n}$, and BR^2 = CR^2 — CB^2 $= \frac{mm}{nn} -$
Therefore in the following table, againſt the name
of the bodies in the firſt column, you have th
ſines of incidence and refraction in the ſecond
the ſquare of BR or the refracting power in th
third, their denſity or ſpecific gravity in the fourth
and their ratio in the fifth column.

Fig. 48.

Bodies	Refraction m	n	Refracting power	Denſity	the Ratio
Pſeudo topaz	23	14	1.699	4.27	0.39
Air	3201	3200	0.000625	0.0012	.52
Glaſs of Antimony	17	9	2.568	5.28	.48
A Selenitis	61	41	1.213	2.252	.54
Glaſs	31	20	1.4025	2.58	.54
Rock Cryſtal	25	16	1.445	2.65	.54
Iſland Cryſtal	5	3	1.778	2.72	.65
Sal Gemmæ	17	11	1.388	2.143	.64
Alumn	35	24	1.1267	1.714	.65
Borax	22	15	1.1511	1·714	.67
Niter	32	21	1.345	1.9	.70
Vitriol	303	200	1.295	1.715	.75
Ol. Vitriol	10	7	1.041	1.7	.61
Rain Water	529	396	0.7845	1.	.78
Gum Arabic	31	21	1.179	1.375	.85
Spt. Wine	100	73	0.8765	0.866	1.01
Camphire	3	2	1.25	0.996	1.25
Oil Olive	22	15	1.1511	0.913	1.26
Linſeed Oil	40	27	1.1948	0.932	1.28
Spt. turpentine	25	17	1.1626	0.874	1.32
Amber	14	9	1.42	1.04	1.36
Diamond	100	41	4.949	3.4	1.45

This table of Sir J. Newton's, made from a great many experiments, ſhews that theſe bodies that differ from a conſtant ratio, are ſuch as abound more or leſs with oily ſulphureous particles. And that ſuch oily ſulphureous ſubſtances have a greater refractive power than others, in proportion as they are of a more ſulphureous nature.

Cor. 1. *The ſquare of* BR *is as the denſity of the medium.*

Fig. 48. Cor. 2. *Refraction out of a thinner medium into a thicker, is made towards the perpendicular; and the contrary.*

Cor. 3. *When light paſſes out of a thicker into a thinner medium, if the obliquity of the incident ray be ſo great, that the ſine of its incidence has a greater proportion to the radius, than the ſine of incidence to the ſine of refraction; that is, when the ſine of refraction would be greater than the radius, which is impoſſible. Then the light will not paſs into that medium, but be reflected back. And therefore light will not paſs out of glaſs into air, above 42 degrees of incidence; but is reflected back into the glaſs.*

PROP. VI.

50. *Let* AGFO *be any denſe medium,* AE *perp. to* AO, HO *an incident ray, making the angle* HOF *infinitely ſmall,* OB, OC *the refracted rays, being the leaſt and moſt refrangible. Draw* Bc *parallel to* AO, *cutting* OC *in c; thro' c draw ace parallel to* AE; *then in any other denſe medium* AGH, *if the leaſt refrangible rays (of the incident ray* HO*) paſs to* D; *then drawing* De *parallel to* AO *to cut ae in e, the moſt refrangible rays will paſs thro' e.*

Let the ray OE cut D*e* in *r*, and if HO or OA be the velocity of the incident ray, then (Prop. I. Cor. 5.) OB, OC, OD, OE will be the velocity of the rays going to B, C, D, E; and (Prop. V. Cor. 1.) $AB^2 : AD^2$: : denſity of the firſt medium : denſity of the ſecond : : $AC^2 : AE^2$. And AB : AC : : AD : AE; whence AC : CB : : AE : ED. And by ſimilar triangles, B*c* : AO : : (CB : CA : : ED : EA : :) D*r* : AO; whence D*r* = B*c* = D*e*, and *r* falls upon *e* in the line *abe*. Therefore in all mediums, where the angle of incidence is a right angle, the extreme refracted rays will cut the parallels AE, *ae*, in the points B, *c*, and D, *e*, making B*c*, D*e* alſo parallel to one another, and to the refracting ſurface OA.

Fig. 50.

But this does not hold good when the angle of incidence is not a right angle.

Cor. 1. *If the medium is ſuch, that the mean refrangible rays, are refracted at an angle of* 45° *with* OA, *then the rays will be the moſt diſperſed by refraction.*

For the angle DOE will be greateſt when AO makes 45 degrees with a line drawn from O, biſecting D*e*.

Cor. 2. *It does not hold that the ſines of refraction of the heterogeneous rays, are proportional,* S.B : S.C : : S.D : S.E.

For that proportion in inconſiſtent with this, AB : AC : : AD : AE.

Cor. 3. *Neither does this conſtruction hold good, in one and the ſame medium, for different incidencies.*

For if the heterogeneous rays at incidence 90, paſs thro' B, *c* ; at leſs incidencies they will not paſs thro' D, *e* ; the line D*e* growing perpetually leſs, as the angle AOD grows bigger. For in this caſe, it is, S.B : S.C : : S.D : S.E, which is inconſiſtent with the proportion AB : AC : : AD : AE, on which the equality of B*c*, D*e*, depends.

SCHOLIUM.

If the denſe medium AGHA be glaſs, then if OA be 40, A *a* will be 1 ; as may be calculated from having OA given, and the ſines of incidence and refraction. And this may ſerve for a ſtandard, whereby to meaſure the refractions of other mediums.

Fig.

PROP. VII.

52. *If homogeneal rays be refracted by a priſm* ABC, *the inclination of the incident and emergent rays is leaſt, when the refraction is equal on both ſides,* APH = BSR.

51. In order to prove this, let the angle *b*I*g* = *g*I*d*, and let *b*I, *g*I, *d*I, be three homogeneal rays, paſſing out of a denſer into a rarer medium, and refracted at I, in the lines IB, IG, ID; draw the ſines *bn*, *go*, *dp*, and BN, GO, DP. Now it is known, that in ſmall arches, theſe arches are nearly as their ſines, and the increaſes of the arches as the increaſes of the ſines; but as the arches increaſe, that increaſe has a greater ratio to the increaſe of the ſine; and ſtill greater and greater as the arches become bigger. Now ſince the ſines *bn*, *go*, *dp* have a given ratio to BN, GO, DP, and their increments to their increments. And ſince AB is greater than *ab*, therefore, putting *inc.*OG for increaſe of OG, &c. we have $\frac{dg}{inc.og}$ greater than $\frac{bg}{inc.nb}$ or $\frac{dg}{inc.nb}$, and *inc.nb* greater than *inc.og*. And $\frac{DG}{inc.OG}$ much greater than $\frac{BG}{inc.NB}$, that is, $\frac{DG}{inc.og}$ much greater than $\frac{BG}{inc.nb}$, and $inc.nb \times DG$ much greater than $inc.og \times BG$. Divide by *inc.nb*, and *inc.og* (which are not in ſo great a ratio), and then DG is greater than BG. Whence the ſum of the refractions AB, AD, is greater than twice the refraction AG, into the rare medium.

52. Now let the ray PS be refracted into the lines PH, SR making the ∠ APH = BSR. Alſo let

the ray PQ be refracted into PI and QL. Draw PON perp. to CA, and QO, SN perp. CB. Produce RS, HP to V, and IP, LQ to T. Then $<OPQ + OQP = C = < NPS + NSP =$ 2NPS, for O and C, or N and C, make two right angles. Therefore the angles OPQ, OPS, and OQP have equal differences, and their angles of refraction are OPT, OPV, and OQT. Therefore, by what was ſhewn before, $OPT + OQT$ is greater than 2OPV or $NPV + NSV$; and taking away the equal angles $OPQ + OQP = NPS + NSP$, and there remains $QPT + PQT$ greater than $SPV + PSV$, that is, DTQ greater than FVS, and therefore the inclination RVF is leſs than any other inclination LTD. Fig. 52.

Cor. 1. *The greater the obliquity of the ray* IP *entering into the priſm, the greater will be the inclination of the incident and emergent rays,* DTL.

This appears from what was proved in *fig.* 51, where the greater the arch is, the greater is the ratio thereof to the ſine.

Cor. 2. *If a priſm be placed near the hole in a window ſhut, for a beam of the ſun to paſs thro', ſo as to throw the image upwards; the loweſt place of the image where it appears ſtationary (in turning the priſm ſlowly about), is when the incident and emerging rays make equal angles on each ſide the priſm.*

Cor. 3. *There are two poſitions of the priſm, where the image will have the ſame place, and that is where the ray paſſing thro' it, cuts the contrary ſides at the ſame angles.* 53.

For if IPQD be the progreſs of the ray, and if $<CGF: = < CPQ$, and $CFG = CQP$. Then the ſum of the refractions of the ray FG, will be equal to the ſum of the refractions of the ray PQ. Therefore if the priſm be turned about the axis,

Fig. till the angle IPA be equal to DQB; then the ray
53. will paſs along FG and go to D, as before it did
in going along PQ.

52. Cor. 4. *When the angles of incidence and refraction are equal on both ſides the priſm; the angle of refraction* NPS *is equal to half the angle* C *of the priſm.*

For we proved before that C is equal to 2NPS.

PROP. VIII.

54. *Heterogeneous rays being refracted by a priſm* ABC, *the more oblique the incident ray* IP *is, the leſs will b the ſeparation of the rays* SL., Q*l*.

Inſtead of a more tedious method of demonſtration, I ſhall compute the progreſs of the rays trigonometrically, the angle C being 60°, and the ſines of incidence and refraction out of air into glaſs being as 106 to 68 and 69.

1. Let the angle of incidence API be infinitely ſmall, then it will be

106 —— 2.0253059	106 —— 2.025305
68 —— 1.8325089	69 —— 1.838849
Rad. —— 10.	Rad. —— 10.
S.39 54 —— 9.8072030	S.40 37 —— 9.813543
the angle OPQ.	the angle OPS.

Theſe angles taken from (C) 60, leave 20 6 and 19 23 for the angles of incidence at Q and S Therefore again,

68 —— 1.8325089	69 —— 1.838849
106 —— 2.0253059	106 —— 2.025305
S.20 6 — 9.5361286	S.19 23 — 9.520989
11.5614345	11.546295
S.32 23½ — 9.7289256	S.30 39 — 9.70744

dif. $\dfrac{30\ \ 39}{1\ \ 44\frac{1}{2}}$ = the angle of diſſipation betwee the rays SL, Q*l*.

2. L

2. Let *l*Q be the incident ray; then ſince the moſt refrangible ray will return thro' *l*QP to I, making the angle API infinitely ſmall. Therefore we have nothing to do, but trace the leaſt refrangible ray thro' *l*QRH. Therefore Fig. 54.

106 —— 2.0253059
69 —— 1.8388491
S. 32 $23\frac{1}{2}$ — 9.7289256
11.5677747
S. 20 $24\frac{1}{2}$ — 9.5424688
the angle RQO.

Then 20 $24\frac{1}{2}$ taken from 60 leaves 39 $35\frac{1}{2}$ for the angle of incidence at R, or QRO, therefore

69 —— 1.8388491
106 —— 2.0253059
S. 39 $35\frac{1}{2}$ — 9.8043520
11.8296579
S. 78 15 — 9.9908080

And 78 15 taken from 90, leaves 11 45 for the angle made by RH and PI, for the ſeparation of the rays at P or R, far exceeding that at Q or S.

3. Let the angles at P, Q be equal, being each equal to 60; whence the angle of incidence is 30. Then

68 —— 1.8325089
106 —— 2.0253059
S.30 —— 9.6989700
11.7242759
S.51 $12\frac{1}{2}$ — 9.8917670

69 —— 1.8388491
106 —— 2.0253059
S.30 —— 9.6989700
11.7242759
S.50 11 — 9.8854268

51 $12\frac{1}{2}$
50 11
diff. 1 $1\frac{1}{2}$

And 1 $1\frac{1}{2}$ doubled gives 2 : 03 the whole ſeparation by both ſides of the priſm, when the angles

Fig. at P and Q are nearly equal, and the rays come on
54. any ſide.

So that when the rays fall very obliquely, the ſeparation of the heterogeneous rays is only 1 44½. When their incidence and emergence are about equal, it is 2 03. And when their incidence is ſo near a perpendicular, that their emergence is the moſt oblique, it is 11 45.

PROP. IX.

55. *If* R *be the radiant point,* RP *perpendicular to the refracting plane ſurface* PA. *Then if* RA *be the incident ray,* AH *the refracted ray, cutting* RP *in* F; *then* F *will be focus, ſo that* FA : RA : : S.*incidence* : S.*refraction.*

For the angle PRA = angle of incidence, and PFA = angle of refraction, and in the triangle ARF, FA : RA : : S.FRA : S.PFA : : S.angle of incidence : S.angle of refraction.

Cor. 1. *As the ſine of refraction : to the ſine of incidence : ſo the incident ray to the refracted ray.*

Cor. 2. *If rays diverging from* R, *diverge from* F, *after refraction; then rays converging to* F *will converge to* R *after refraction.*

Cor. 3. *Parallel rays falling on a refracting plane, are refracted into parallel lines.*

For R and F being at an infinite diſtance; all rays proceeding from R will be parallel to one another, and all rays proceeding from F will alſo be parallel.

Cor. 4. *If rays diverging from* R, *diverge from* F *after refraction; then changing the mediums, the rays diverging from* F, *will diverge from* R, *after refraction.*

Cor. 5. *If rays* RA, R*a*, *falling near* P, *be refracted from the focus* F; *then* FP : RP : : S.*incidence* : S.*refraction*. Fig. 56.

For then FA = FP, and RA = RP, nearly.

Cor. 6. *If* R *is an object under water, and the eye moves from* C *to* D *and then to* H, *&c. the object will ſeem to riſe in the perp.* RP, *till at laſt it vaniſhes in* P. *Hence water looks deepeſt when looked at perpendicularly; but it appears not then to be ſo deep as it is, by a fourth part.* 57.

For when the angle of incidence ARP is 48 : 40, the angle of refraction AFP becomes 90. And at C, the apparent place of R will be at F, ſo that FP : RP : : 3 : 4, or PF = $\frac{3}{4}$ PR.

PROP. X.

If AV *be any refracting ſpherical ſurface,* C *the center,* R *the radiating point,* F *the focus after refraction; then* RA × *ſine of incidence* : CR × *ſine of refraction* : : AF : CF. 58. 59. 60. 61.

Let RA be the incident ray, AG the refracted one. Draw FH parallel to CA, cutting RA in H. Then $<$ RHF = RAC = angle of incidence, and AFH = FAC = angle of refraction. Put *m*, *n*, for the ſines of incidence and refraction. Then in the triangle AFH,

S.RHF or AHF (*m*) : S.AFH (*n*) : : AF : AH.

And by ſimilar triangles, AR : CR : : AH : CF.

and ex equò, $m \times$ AR : $n \times$ CR : : AF : CF.

Cor. 1. *On the contrary, if the rays go the contrary way in the line* GA *or* FA, *they will be refracted to* R; *and* R *and* F *will be conjugate foci.*

Cor. 2. *If rays diverging from* R, *converge to* F *then changing the mediums, rays converging to* R *wi* *diverge from* F. *Or if rays diverging from* R, *diverge from* F, *then changing the mediums, rays converging to* R, *will converge to* F, *after refractio* *Which ſaves the labour of drawing more figures.*

PROP. XI.

58. *If* AV*a be a very ſmall part of a refracting ſpherical*
59. *rical ſurface,* C *the center,* R *the radiating point*
60. *object,* F *the focus after refraction;*
61. *Then the diſtance of the object and vertex* RV × *ſi*
of incidence :
Diſtance of the object and center RC×*ſine of refraction*
Diſtance of the vertex and focus VF :
Diſtance of the center and focus CF, *nearly.*

Let RA be the incident ray, AG or AF the refracted ray; then ſince A is very near the vertex V, $RA = RV$, and $AF = VF$ nearly. And (Prop. X putting m, n for the ſines of incidence and refraction, $m \times AR : n \times CR :: AF : CF$; that is, $m \times RV$ $n \times CR :: VF : CF$.

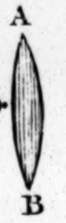
Cor. 1. *If* m, n, *be the ſines of incidence and refraction, then* $m \times VR \overset{\backsim}{+} n \times CR : m \times VR :: CV$ VF.

This follows from the Prop. by compounding o dividing.

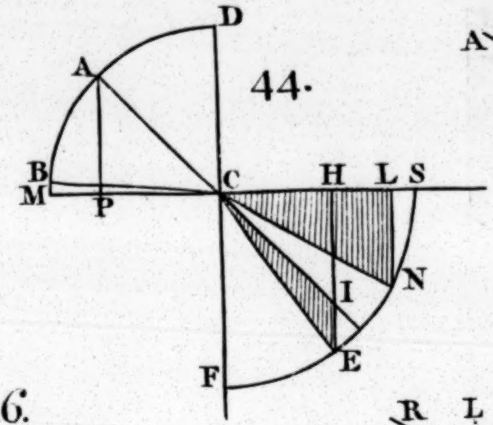
Cor. 2. *If the rays be turned back again in the li* GA, *they will be refracted to the focus* R; *whi* *ſupplies the place of more figures.*

Cor. 3. *If* R *and* F *lye on the ſame ſide of the refracting ſurface; rays diverging from* R *will diverg* *from* F; *if they lie on different ſides, they converg* *to* F.

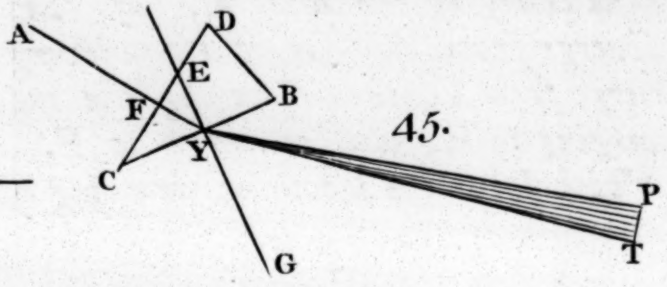
Fig 43.



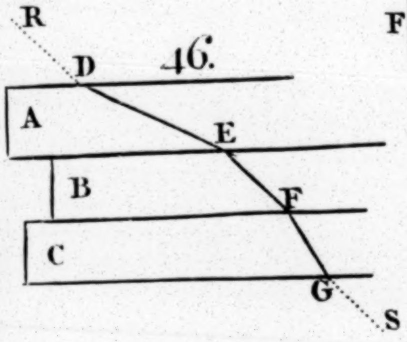
44.



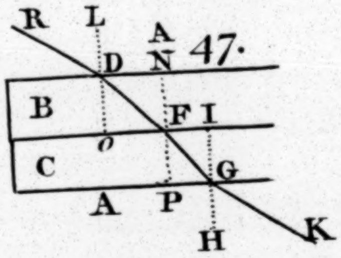
45.



46.



47.



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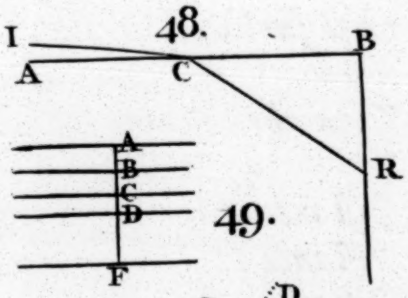

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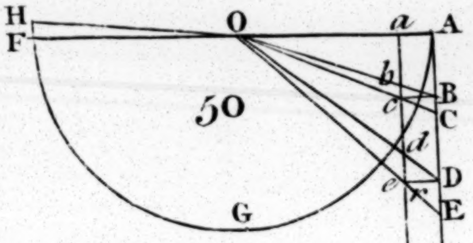

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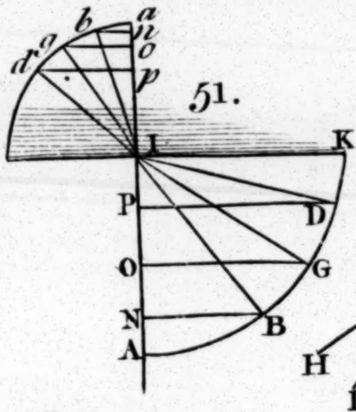

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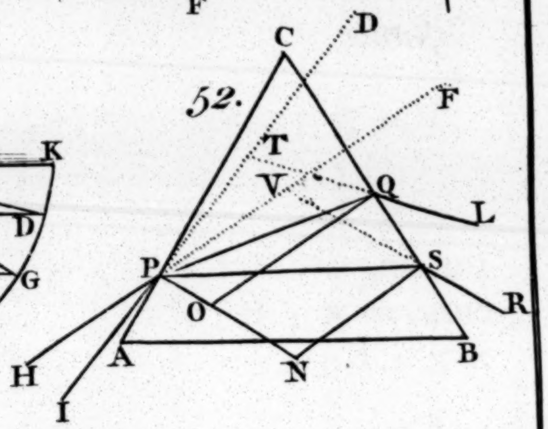

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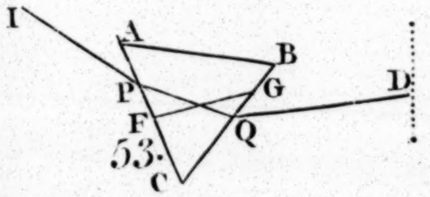

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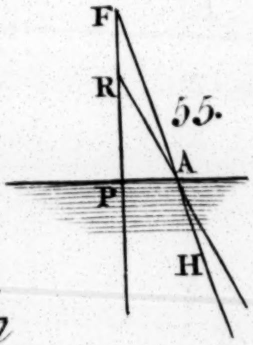

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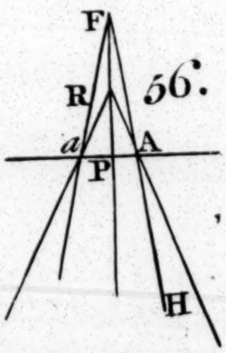

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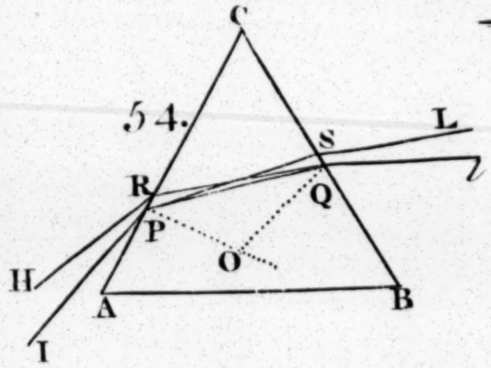

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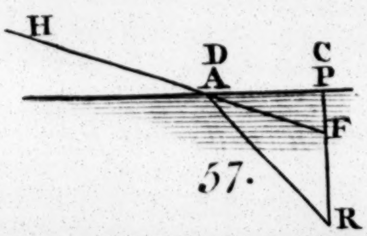

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PROP. XII.

If parallel rays RA, RV *fall upon any refracting* 62.
ſpherical ſurface AV, *whoſe center is* C; F *will be* 63.
the focus of the refracted rays, ſo that, the ſine of incidence : ſine of refraction : : AF : CF.

Draw the radius CA, then RAC or ACF is the angle of incidence (or its ſupplement), and FAC the angle of refraction. Let m, n, be the ſines of incidence and refraction. Then in the triangle ACF, S.ACF (m) : S.CAF (n) : : AF : CF.

Cor. 1. *If the rays be turned back in the line* GA, *and refracted at* A, *they will emerge parallel.*

Cor. 2. *If* AV *be any refracting curve,* AC *perp. to the curve in* A; *and* CF *parallel to* RA; *and if* RA *be the incident ray,* AF *the refracted ray; then* AF : FC : S.*incidence* : S.*refraction.*

PROP. XIII.

If parallel rays RA, RV, *fall on a very ſmall part* 62.
of a refracting ſpherical ſurface A*a*, *whoſe center is* 63.
C; *and if* F *be the focus, then ſine of incidence : ſine of refraction* : : VF : CF, *nearly.*

For as AV is a very ſmall arch, AF = VF nearly, therefore (Prop. XII), S.incidence : S.refraction : : AF (VF) : CF.

Cor. 1. *If* m, n, *be the ſines of incidence and refraction; then* $m - n : m$: : *radius* CV : VF *the principal focal diſtance from the vertex.*

For $m : n$: VF (VC + CF) : CF; and by diviſion, $m - n : m$: : VC : VF.

Cor.

Fig. Cor. 2. *The focus of parallel rays refracted by a*
62. *convex ſpherical ſurface of glaſs is diſtant from the*
63. *vertex near three times the radius.*

For m is to n, nearly as 3 to 2, and $m - n$ to m as 1 to 3.

Cor. 3. *As* $m - n$ *the difference of the ſines* : *to* n *the leſſer* : : *as* CV *the radius* : *to* VF — VC, *or* CF *the diſtance of the focus from the center.*

PROP. XIV.

64. *If* AV *be a ſmall portion of a refracting ſpherical*
65. *ſurface,* C *the center,* t, T *the principal foci, or the foci of parallel rays coming out of one medium into the other. And if* R *be the radiating point,* F *the focus; then it will be,* Rt : RV : : RC : RF; *or the contrary, changing the focus for the radiating point.*

Let m, n be the ſines of incidence and refraction, d = RV, and r = radius CV. Then (Cor. 1. Prop. XIII.) $m - n : m :: r : \frac{rm}{m - n} = VT$. And on the contrary $n - m : n :: r : \frac{rn}{n - m} = Vt$, or rather $Vt = \frac{rn}{m - n}$, and VT — VC or CT = $\frac{rm}{m - n} - r = \frac{rn}{m - n}$, therefore $Vt = CT$, and $VT = tC$.

But (Prop. XI. Cor. 1.) $md - n \times \overline{d + r} : md :: r : VF$ (in fig. 64.), or $md - nd - nr : mr :: d : VF$; and dividing by $m - n$, $d - \frac{nr}{m - n} : \frac{mr}{m - n} :: d : VF$; that is, RV — V$t$ or Rt : VT or tC : : RV : VF. And compounding, Rt : Rt + tC (RC) : : RV : RV + VF (RF). And (in fig. 65.)

it is $md - nd + rn : md :: r : VF$. Whence RV Fig.
$+ Vt : VT$ or $Ct :: RV : VF$, &c. 64.
65.

Cor. 1. $Rt : tC$ *or* $VT :: RV : VF$.
From the demonſtration.

Cor. 2. $Rt : tC :: CT : TF$.
For (Cor. 1.) $Rt : tC :: RV : VF :: RV - Rt$ (tV or CT) : $VF - tC$ or $VF - VT$ (TF).

Cor. 3. $Rt : tV :: VT : TF$.
For (Cor. 1.) $Rt : TV :: RV : VF :: RV - Rt$ (tV) : $VF - VT$ (TF).

Cor. 4. *If* T, *t be the principal foci, of rays paſſing out of one medium into the other*; *then* $CT = Vt$.
From the demonſtration.

PROP. XV.

If AV *be a ſmall portion of a refracting ſpherical ſurface*; C *the center*, R *the radiating point or the focus of the incident rays*, F *the focus of the refracted rays. And if* m = S.*incidence*, n = S.*refraction*, $RV = d$, $CV = r$, $VF = f$; *then* $f = \frac{mdr}{\overline{m - n} \times d - nr}$; *where if the rays converge, or* AV *be concave*; d *or* r *will be negative.* 66.

For ſince the arch AV is very ſmall, $RA = RV = d$, $AF = VF = f$. Draw FH parallel to CA, cutting RA in H. Then by ſimilar triangles RC ($d + r$) : CF ($f - r$) :: RA or RV (d) : $AH = \frac{df - dr}{d + r}$. Then in the triangle AFH, S.AHF or CAH (m) : S.AFH or CAF (n) :: AF or VF (f) : AH $\left(\frac{df - dr}{d + r}\right)$. Whence $nf = \frac{df - dr}{d + r} m$, and

Fig. 66. and $ndf + nrf = dmf - dmr$, or $mdf - ndf - nrf = mdr$, therefore $f = \frac{mdr}{md - nd - nr}$.

Cor. 1. $f = \frac{mdr}{\overline{m-n}\,.\,d - nr} = \frac{mdr}{md - n \times \overline{d+r}}$.

Cor. 2. *If rays fall on a concave refractor*, $f = \frac{-mdr}{\overline{m-n}\,.\,d + nr}$.

Cor. 3. *If the rays converge to* R, $f = \frac{mdr}{\overline{m-n}\,.\,d + nr}$.

For $f = \frac{-mdr}{\overline{m-n} \times -d - nr}$.

Cor. 4. *If it be a plain ſurface*, $f = \frac{-md}{n}$.

For it is $f = \frac{mdr}{-nr} = -\frac{md}{n}$, r being infinite.

Cor. 5. *If the rays be parallel*, $f = \frac{m}{m-n}r$.

For then d is infinite, and $f = \frac{mdr}{\overline{m-n}\,.\,d}$.

SCHOLIUM.

From this general ſolution it follows, that when f the focal diſtance comes out affirmative, it muſt be taken directly forward from V, but when it is negative, it muſt be taken backward towards R.

In the ſame manner that f is found from having d, and r, given; either d or r may be found from having the reſt given. For example, let r be ſought. Multiply the equation, $f = \frac{mdr}{md - nd - nr}$, by the denominator $md - nd - nr$, and we ſhall Fig. have $mdf - ndf - nrf = mdr$; and tranſpoſing, 66. $mdr + nfr = mdf - ndf$, whence $r = \frac{mdf - ndf}{md + nf}$. And this ſhews that when m is greater than n, r comes out affirmative, and that the refractor is convex towards the object. But when n is greater than m, r will be negative, and therefore 'tis concave to the object.

Again, let d be ſought, having f, r, given. From the former equation, $mdf - ndf - nrf = mdr$; by tranſpoſing, $mdf - ndf - mdr = nrf$, and $d = \frac{nrf}{mf - nf - mr}$. And from hence it appears, that if mf be greater than $nf + mr$, then d is affirmative; that is, R is on the convex ſide of the refractor. But when $nf + mr$ is greater than mf, then d comes out negative, and R is on the concave ſide of the refractor. And if mf be $= nf + mr$, then d will be infinite, and F is the focus of parallel rays. 67.

It may be further obſerved, that in all caſes where the rays converge to F, if a circle BD be deſcribed with any radius from the center F, to cut the refracted rays AF; theſe rays, paſſing thro' the meniſcus AVDB, will go to the ſame focus F. For theſe rays, paſſing perpendicularly thro' the refracting ſurface BD, go right forward. And for the ſame reaſon, rays diverging from F, will go to R as before.

PROP. XVI.

If parallel rays RA, *t*V *fall upon a ſmall portion of a ſphere, and be refracted by the firſt ſurface to* T; *they will be refracted by the ſecond ſurface to* F, *the middle of* DT. 68.

Let the rays RA, FG meet in H; then ſince the cord AG cuts the curve at A, G in equal angles, Fig. 68. gles, the refractions will be equal at A and G; whence the triangle AHG is equiangular, and equilateral. But FT being parallel to AH, the triangle GFT is ſimilar to AHG; whence GF = FT. But GF being = DF very near, therefore DF = FT.

Cor. *If parallel rays fall upon any part of a ſphere; and* T *be the focus after the firſt refraction;* F *will be the focus after the ſecond, ſo that* GF = FT.

PROP. XVII.

69. 70. *In any lens* BH, *if two radii* CA, *ca*, *be drawn parallel to one another, from the centers* C, *c*, *of the two ſurfaces; and* A*a be drawn cutting the axis in* D. *Then every ray that paſſes thro'* D, *ſhall have its incident and emergent rays parallel.*

For by reaſon of the parallels CA, *ca*, the triangles CAD, *cad*, are ſimilar; therefore CD is to *c*D, as CA to *ca*, that is in a given ratio; and therefore the point D is invariable. And further the angles at A and *a* being equal, the line A*a* cuts the two ſurfaces in the ſame angle, and therefore if the ray A*a* be ſuppoſed to go both ways, the refractions will be equal and contrary; that is, they make equal angles with A*a*, and lie contrary ways, and therefore are parallel.

Cor. 1. *Hence in double convex and double concave lenſes, the point* D *lies within the lens; in a plano convex or plano concave, it is at the convex or concave ſurface; but in a meniſcus, it lies out of the lens, nearer to the greateſt curvity. And in a ſphere, it is at the center.*

Cor. 2. *The courſe of a ray, paſſing thro'* D, *very near the axis, may be taken for a ſtrait line, without ſenſible error.*

Cor. 3. *The point* D *is ſo placed, that* CG : *c*F : : Fig.
DG : DF. 69.
For CA or CG : *ca* or *c*F : : CD : *c*D : : CG — 70.
CD (DG) : CF — CD : (DF).

Cor. 4. *As the ſum of the radii (or the difference in a meniſcus) : to the leſſer radius : : ſo the thickneſs of the lens : to the diſtance of* D *from the vertex of greater curvature.*

For ſince CG : *c*F : : DG : DF, by compoſition (or diviſion) CG ± *c*F : *c*F : : DG ± DF (FG) : DF.

Cor. 5. *If* A*a be any ray paſſing thro'* D; *then the tangents to the lens at* A *and a, will be parallel.*

For the angles CA*a* and *ca*A being equal, their complements DAG and D*a*F are equal; and therefore the tangent at A is parallel to the tangent at *a*. And hence

Cor. 6. *For every point* A *of the lens, there is ſome ray or other* A*a, whoſe incident and refracted rays will be parallel; and that is the ray paſſing thro'* D.

For the tangents at A and *a* being parallel, it is the ſame thing as if the ray was tranſmitted thro' a medium contained between two parallel ſurfaces, in which caſe (by Cor. 2. Prop. III.) the incident and emergent rays will be parallel.

SCHOLIUM.

The point D is by optic writers called the *Center* of the lens.

PROP. XVIII. *Prob.*

*Having the lens a*V, *and the radiant point* R 71.
given; to find the focus F, *after the refraction of* 72.
both the ſurfaces.

Let *m*, *n*, be the ſines of incidence and refraction, and make *m* × R*v* — *n* × R*c* : *m* × R*v* : :

Fig. the radius $vc : vf$, and f is the focus after the firſt
71. refraction at av. Again $vf - vV = Vf$; then
72. make as $m \times Cf - n \times Vf : n \times Vf :: \text{radius } CV : VF$, and F is the focus after the ſecond refraction at AV. And thus you may proceed to find the focus after refraction by a third, or fourth ſurface, &c.

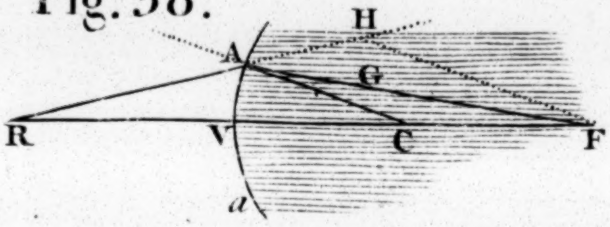
For ſince $m \times Rv - n \times Rc : m \times Rv :: vc : vf$, by diviſion, $m \times Rv : n \times Rc :: vf : vf - vc$ or cf. And therefore (by Prop. XI.) f is the focus of the rays refracted at av. Alſo ſince $m \times Cf - n \times Vf : n \times Vf :: CV : VF$; therefore, $n \times Vf : m \times Cf :: VF : CV + VF$ or CF. And $Vf : Cf :: \frac{VF}{n} : \frac{CF}{m} :: m \times VF : n \times CF$. And therefore (Cor. 2. Prop. XI.) rays converging to f and refracted by AV, will converge to the focus F. This method of demonſtration is general, and is applicable to any caſe.

Cor. *If the incident rays are parallel; make as* $m - n : m :: vc : vf$, *and* f *is the focus after refraction at the firſt ſurface. And find* F *as before, for the focus after the ſecond refraction.*

For in this caſe Rv and Rc are equal.

PROP.

Fig. 58.



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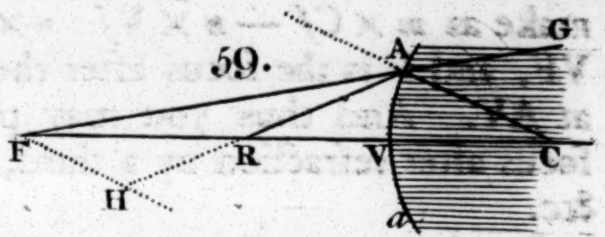

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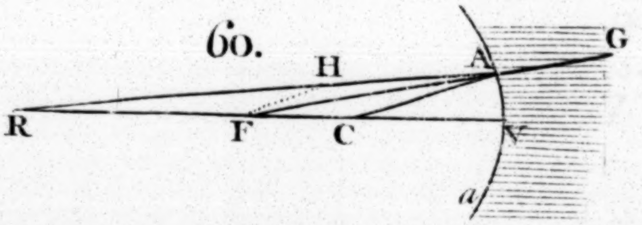

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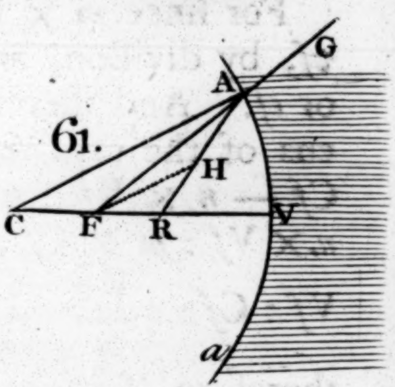

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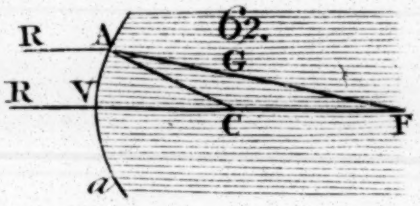

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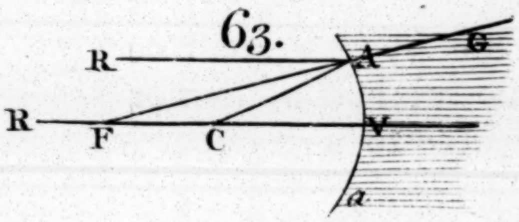

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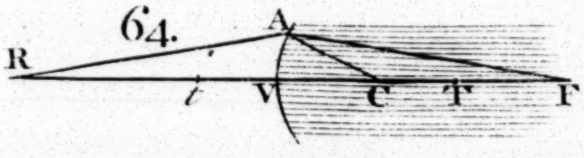

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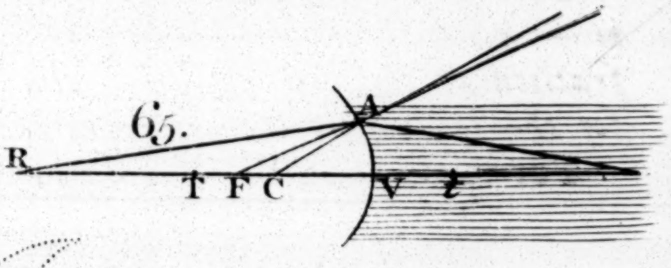

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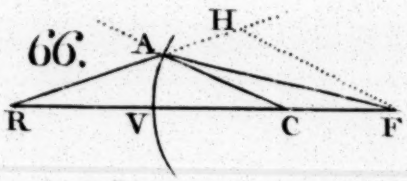

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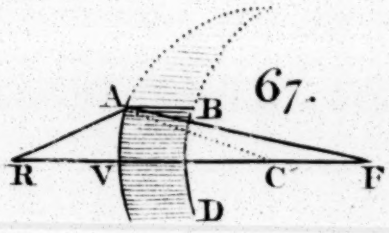

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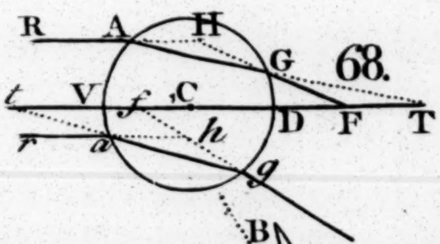

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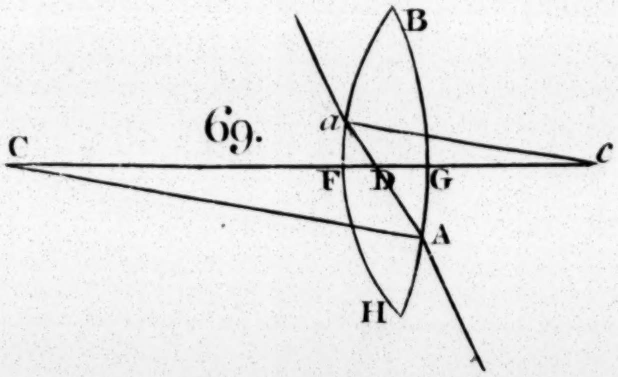

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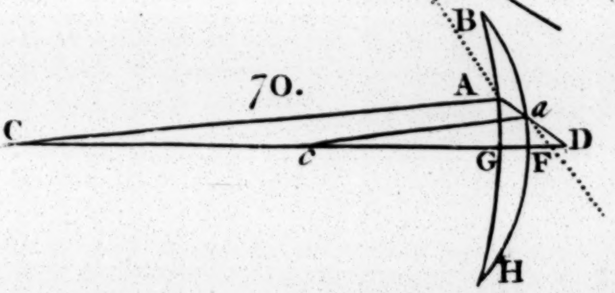

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Fig.

PROP. XIX.

Let parallel rays Sa, Rv, *be refracted by the lens* 71.
Av; *ſuppoſe the ſines of incidence and refraction as* 72.
m to n; *and put the radius of the ſegment towards the object* $cv = r$, *the radius of the ſegment from the object* CV = R, *the thickneſs of the lines* vV $= t$, *the principal focal diſtance* VF $= f$. *And* $p = \frac{n}{m - n}$, $q = \frac{m}{m - n}$; *therefore* $mp = nq$. *Then*

$$f = \frac{Rn \times \overline{qr - t}}{m \times \overline{R + r} - t \times \overline{m - n}};$$ *wherein if av, or* AV *be convex the contrary way, then r or* R *is negative.*

For (Prop. XVIII. Cor.) $m - n : m :: vc\ (r) : vf = \frac{mr}{m - n} = qr$; and $Vf = qr - t$. Again $m \times Cf - n \times Vf : n \times Vf :: CV : VF$; that is, $m \times \overline{R + qr - t} - n \times \overline{qr - t} : n \times \overline{qr - t} :: R : f$

$$= \frac{Rn \times \overline{qr - t}}{mR + mqr - mt - nqr + nt} =$$

$$\frac{Rn \times \overline{qr - t}}{mR + \overline{m - n} . qr - \overline{m - n} . t} =$$

$$\frac{Rn \times \overline{qr - t}}{mR + mr - t \times \overline{m - n}} = \frac{qr - t}{q \times \overline{r + R} - t} pR.$$

And a like calculation may be made for the concave, fig. 72.

Cor. 1. *If* $t = 0$, *or inconſiderable, then* $f = \frac{pRr}{R + r}$, *and is the ſame on either ſide of the lens.*

For then $f = \frac{nqRr}{m \times \overline{R + r}} = \frac{mpRr}{m \times \overline{R + r}} = \frac{pRr}{R + r}$.

Fig. 71. 72. Cor. 2. *If* $t = 0$, *and* av *be a plane ſurface, then* $f = \frac{n}{m - n} R$.

For then $r =$ infinity, and $f = \frac{pRr}{r} = pR = \frac{n}{m - n} R$. And

Cor. 3. *If* $t = 0$, *and* AV *a plane ſurface,* $f = \frac{n}{m - n} r$.

Cor. 4. *If* $t = 0$, *and* $R = r$, *then* $f = \frac{n}{m - n} \times \frac{1}{2} r$.

Cor. 5. *The principal focal diſtance made by one refraction, is* $\frac{mr}{m - n}$.

For by the demonſtration $vf = \frac{mr}{m - n}$.

Cor. 6. *If a luminous body be placed in the principal focus of a lens, it will caſt a light to a great diſtance.*

For the rays emerging nearly parallel, will not unite till at a great diſtance.

SCHOLIUM.

In glaſs, $m = 3$, and $n = 2$ nearly; whence $p = 2$, and $q = 3$. Therefore from this Prop and its corollaries, may be deduced the following rules peculiar to glaſs lenſes.

1. In a double convex glaſs of equal radii, the principal focus is diſtant the radius of the ſphere; or the focal diſtance is equal to the radius

2. In a ſphere, the principal focus is diſtant half the radius; of an hemiſphere $\frac{4}{3}$ the radius, when

the convex ſide is expoſed to the rays; and twice Fig.
the radius, when the plane ſide is expoſed. 71.

3. In a plano convex, the diſtance of the principal focus is equal to the diameter, when the plane ſide is expoſed to the rays; but to the diameter — $\frac{2}{3}$ the thickneſs, when the convex ſide is expoſed. 72.

4. In a double convex glaſs of different ſpheres, the diſtance of the principal focus is $\frac{2Rr}{R+r}$.

5. In double concaves of equal radii, the principal focus is diſtant the radius of the ſphere, and is virtual.

6. In a plano concave, the principal focus is diſtant twice the radius, and is virtual.

8. In a double concave glaſs of different ſpheres, the diſtance of the principal focus is $-\frac{2Rr}{R+r}$, and is virtual.

9. In a meniſcus, the diſtance of the principal focus is $\frac{2Rr}{r-R}$, the concave ſide being expoſed to the rays.

Fig.

PROP. XX.

71. *If rays iſſuing from the radiant point* R, *be refracted by the lens* Av; *let* m, n = *ſines of incidence and*
72. *refraction,*

d = Rv, *the diſtance of the radiant point.*
r = cv, *the radius of the nearer ſurface.*
R = CV *the radius of the further ſurface.*
t = Vv *the thickneſs of the lens.*
f = VF *the focal diſtance.*

$$p = \frac{n}{m-n},\ q = \frac{m}{m-n},\ pm = qn;\ \textit{then}$$

$$f = \frac{mpdr\text{R} - nd\text{R}t + npr\text{R}t}{m\,d \times \overline{r + \text{R}} - mpr\text{R} - dt \times \overline{m - n} + nrt}$$

wherein, if the rays converge, or if av, AV, *be concave the contrary way, then* d, r, *or* R *is negative.*

Or at ſhort, $f = \dfrac{\text{D} - t}{\text{D} - t + q\text{R}}\,p\text{R}$, *where* D = $\dfrac{qdr}{d - pr}$.

For (by Prop. XVIII.) $m \times \text{R}v - n \times \text{R}c : m\times$ Rv : : $vc : vf$; that is, $md - n \times \overline{d + r} : md : : r$: $vf = \dfrac{mdr}{md - nd - nr}$ = D ſuppoſe, and D — t = Vf. Again $m \times \overline{\text{R} + \text{D} - t} - n \times \overline{\text{D} - t} : n\times$ $\overline{\text{D} - t}$: : R : f, or mR + . $\overline{m - n} \times$ D — $\overline{m - n}$. t : nD — nt : : R : f, therefore f =

$$\frac{n\text{DR} - n\text{R}t}{m\text{R} + \overline{m - n}\,.\,\text{D} - \overline{m - n}\,.\,t}.$$

$$\text{But D} = \frac{mdr}{md - nd - nr} = \frac{\frac{m}{m-n}\,dr}{d - \frac{n}{m-n}\,r} = \frac{qdr}{d - pr};$$

therefore f = $\dfrac{nqdr\text{R}}{d - pr}$

Fig. 71. 72.

$$\frac{\frac{nqdrR}{d-pr}-nRt}{mR+\overline{m-n}\times\frac{qdr}{d-pr}-\overline{m-n}\times t}=$$

$$\frac{nqdrR-ndRt+nprRt}{mdR-mprR+\overline{m-n}\,.\,qdr-\overline{m-n}\times\overline{dt-prt}}$$

$$=\frac{mpdrR-ndRt+nprRt}{mdR-mprR+m\times dr-\overline{m-n}\times dt+\overline{m-n}\times prt}$$

$$=\frac{mpdrR-ndRt+nprRt}{md\times\overline{r+R}-mprR-dt\times\overline{m-n}+nrt};$$

becauſe $\overline{m-n}\,.\,q=m$, and $\overline{m-n}\,.\,p=n$. Or at ſhort $f=\frac{D-t}{D-t+qR}pR$, where $D=\frac{qdr}{d-pr}$. And a like computation may be made for the concave, fig. 72.

Cor. 1. *If* t *be nothing or inconſiderable, then* $f=\frac{pdrR}{d\times\overline{r+R}-prR}$, *and is the ſame on both ſides the lens, whilſt the diſtance* d *continues the ſame.*

Cor. 2. *If the lens be of glaſs; then* $f=\frac{6drR-2dRt+4rRt}{3d\times\overline{r+R}-6rR-dt+2rt}$; *or when* $t=0$, $f=\frac{2drR}{d\times\overline{r+R}-2rR}$.

Cor. 3. *In a plano convex, if* $t=0$, *and* AV *a plane ſurface; then* $f=\frac{ndr}{\overline{m-n}\,.\,d-nr}$.

For r vaniſhing, $f=\frac{pdrR}{dR-prR}=\frac{dr}{\frac{d}{p}-r}=\frac{ndr}{\frac{nd}{p}-nr}=\frac{ndr}{\overline{m-n}\,.\,d-nr}$.

Fig. 71, 72. Cor. 4. *If* $t = 0$, *and the lens has equal convexities; then* $f = \frac{ndr}{\overline{m-n}\times 2d - nr}$. *And in glaſs,* $f = \frac{dr}{d-r}$.

Cor. 5. *When* $t = 0$, *in a double concave,* $f = \frac{pdrR}{-d\times\overline{r+R}-prR}$. *And when the radii are equal,* $f = -\frac{dr}{d+r}$, *for glaſs.*

Cor. 6. *In a plano concave, expoſed with the concave ſide to the rays,* $f = \frac{-pdR}{d+pR}$.

Cor. 7. *In a meniſcus, the convex ſide turned to the rays,* $f = \frac{-pdrR}{d\times\overline{r-R}+prR}$.

Cor. 8. *The focal diſtance made by a ſingle refraction is* $\frac{mdr}{\overline{m-n}\,.\,d - nr}$.
From the demonſtration.

Cor. 9. *If the radiant point and the focus be equidiſtant from a lens; they will each of them be diſtant twice the principal focal diſtance.*

For put $d = f = \frac{pdrR}{d\times\overline{r+R}-prR}$, then $d\times\overline{r+R}-prR = prR$, or $d\times\overline{r+R} = 2prR$, whence $d = \frac{2pRr}{r+R}$ = twice the principal focal diſtance, by Cor. 1. Prop. XIX.

Cor. 10. *If the radiant point be nearer the lens, than the principal focus; the rays after refraction will*

will diverge; *if it is in the principal focus, they will* Fig.
merge parallel; *and if it is further off, they will* 71.
onverge. 72.

SCHOLIUM.

Hence if all the quantities m, n, d, r, R, f, xcept one be given, that one may be found, by he reduction of the equation in Cor. 1.

PROP. XXI.

If P, *p*, *be the principal foci of a ſphere or lens* 73.
Av; R *the radiant point*; F *the focus by refraction*; 74.
D *the center of the ſphere or lens*; *then* RP, RD, RF, *are continual proportionals, where* P *and* F *muſt* *ie the ſame way from* R.

Let the focal diſtance DP or Dp = F, the reſt s in the laſt Prop. Then (Cor. 1. Prop. XIX.) Vp or DP $= \frac{prR}{r+R} = F$, and $prR = F \times \overline{r+R}$. And (Cor. 1. Prop. XX.) VF or DF =

$$\frac{dprR}{d\times\overline{r+R} - prR} = \frac{dF\times\overline{r+R}}{d\times\overline{r+R} - F\times\overline{r+R}}$$

$$= \frac{dF}{d-F} = \frac{RD\times DP}{RP},$$ and RP × DF = RD × DP, whence RP : RD :: DP : DF; and compounding, RP : RD :: RP + DP (RD) : RD + DF (RF). And a like computation may be made for the concave, fig. 74.

Cor. 1. RP : RD :: DP : DF; *or* DF $= \frac{RD \times DP}{RD - DP}$.

From the demonſtration.

Cor. 2. RP : PD :: DP : pF; *or* RP × pF = PD².

For

Fig. 73. 74. For (Cor. 1.) RP : RD : : DP : DF ; and by diviſion RP : RD — RP (PD) : : PD : DF — DP (*p*F).

Cor. 3. *p*F *is always reciprocally as* RP ; *and they lie contrary ways from p and* P.

Cor. 4. R *and* F *are conjugate foci to one another; and when they are on the ſame ſide of the lens, the incident and refracted rays diverge both from* R *and* F, *or elſe both converge to* R *and* F. *And the contrary when on different ſides.*

PROP. XXII.

73. 74. *If* A*v* *be a lens or ſphere,* F *the focus of the refracted rays which flow from the radiant point* R, *p* *the principal focus. Then* RF : RD : : DF : D*p*, *the principal focal diſtance.*

For (Prop. XXI.) RF : RD : : RD : RP, and by diviſion, RF : RF — RD (DF) : : RD : RD — RP (PD or *p*D).

Cor. 1. *In a convex, the principal focal diſtance* $Dp = \frac{RD \times DF}{RD + DF}$.

Cor. 2. RF : DF : : DF : *p*F, *or* $pF = \frac{DF^2}{RF} = \frac{DF}{RF} \times DF$.

For ſince RF : RD : : DF : D*p*, by diviſion RF : RF — RD (DF) : : DF : DF — D*p*, (*p*F.)

Cor. 3. *Hence, the focus* F *may be found geometrically thus. Let* P *be the principal focus; with the center* D *and radius* DP, *deſcribe the arch* PG, *cut-*

ting any incident ray RA, *in* G. *Join* DG, *and draw* AF *parallel to it, cutting the axis* RD *in* F, *then* F *is the focus.* Fig. 73. 74.

For RF : RD : : DF : DP ; that is, RF : RA : : DF : AG, therefore the triangles RGD, RAF are ſimilar, and AF parallel to GD. And all rays proceeding from G (or *g*) will emerge parallel to GD.

SCHOLIUM.

In the firſt Corol. we have a good practical rule for finding the principal focus of a lens, having firſt found any other focus by trial. And in Cor. 2. we have another rule tending to the ſame purpoſe, which finds the exceſs of any other focal diſtance found by trial, above the principal focal diſtance. Both theſe are very eaſy in practice.

PROP. XXIII.

The diſtance of the radiating point and its focus, made by a convex lens, that is, RF *is the leaſt poſſible, when they are equidiſtant from the lens.* 73.

For (by Cor. 2. Prop. XXI). RP : PD : : PD : pF. Now if RP $=p$F, then the ſum of the extremes RP, pF is equal to the ſum of the means 2PD, which is conſtant. But in any other caſe (Propor. Prop. IX.) the ſum of the extremes RP, pF is greater than the ſum of the means, that is, greater than the ſum of the extremes when they are equal. And therefore RP $+pf$ is the leaſt when RP $=p$F, and adding Pp, the whole diſtance RF is the leaſt when RP $=p$F, and conſequently when RD $=$ DF.

Cor. *The radiating point and its focus is at the leaſt diſtance, from each other, when each of them is diſtant from the lens, twice the principal focal diſtance.*

For

Fig. 73. For then $RD = RP + PD = 2PD$, and $DF = Dp + pF = 2Dp = 2DP$.

PROP. XXIV.

75. *An oblique pencil of rays* SAGC, *has its focus* G *a little nearer the lens, than a direct pencil* RAFC.

For if DS = DR, then the angle CSA will be ſomething leſs than the angle CRA; and upon that account the rays SA, SC, will be ſooner brought to interſect at G, than the rays RA, RC, do at F. But further, the rays RA, RC, falling very nearly perpendicular to the ſides of the lens AC, HN, are leſs bent, than the rays SA, SC, which fall obliquely on both ſides the lens. For the more oblique they are, the more they are bent, and turned out of their direct way. And conſequently the oblique rays SA, SC will ſooner interſect in G, than thoſe at F. But ſince the angle RDS is generally very ſmall, the difference between DF and DG will likewiſe be very ſmall.

Cor. 1. *The focus of an oblique pencil, by a ſingle refraction is nearly at the ſame diſtance as of a direct pencil.*

For in both caſes the extreme rays will interſect the ray which paſſes thro' the center of the ſurface AC, at the ſame diſtance.

Cor. 2. *This Prop. holds good in a concave lens, and alſo in a meniſcus, as well as in a convex one.*

For in all caſes the more oblique the rays are the more they are bent, and the ſooner they will interſect a direct ray paſſing thro' D the center of the lens, by Cor. 2. Prop. XVII. But I endeavour to avoid drawing a multiplicity of figures.

PROP. XXV.

Fig. 75.

A convex lens collects the rays nearer together, and concave one diſperſes them.

For rays that are refracted at a convex lens, are ent at the points of refraction ſo, that they incline ore towards one another than before: but in con-aves the rays after refraction, being bent the con-ary way, are made to ſeparate more from one an-ther. And this will be ſo whether there be one efracting ſurface, or two ſuch; or if there be more. or every ſucceeding refraction, adds to the effect, y making the rays either converge or diverge ſo uch the faſter.

Cor. 1. *Converging rays are made more converging y a convex lens; and diverging rays are made more iverging by a concave.*

Cor. 2. *A meniſcus has the properties of a convex laſs, when the inner radius is the greater. And of concave, when the inner radius is the leſſer. Ex-pt when the two ſurfaces are concentric; and then belongs to neither; for the rays will emerge parallel.*

Cor. 3. *A lens or ſpectacle glaſs proper for an old erſon, muſt be ſuch as to make the image of an object ppear at that point where he can ſee that object diſ-inctly with his naked eye; and the ſame is true of a oncave, in reſpect of a ſhort-ſighted perſon.*

SCHOLIUM.

From what has been before laid down, the pro-erties of all ſorts of ſingle lenſes may be deduced or practice as follows.

Let R, F, be conjugate foci; P, p the principal foci; then, 76.

Fig. 1. Parallel rays falling on one ſide of a convex
79. lens A; will be refracted to P or *p* on the other
ſide.

2. Parallel rays falling on one ſide of a concave lens A; will by refraction, diverge from P or *p* on the ſame ſide.

3. In a convex lens A, rays diverging from P or *p*; will emerge parallel on the other ſide.

4. In a concave lens A, rays converging to P or *p*; will emerge parallel, going out of the lens on the ſame ſide.

76. 5. In convex lenſes, rays diverging from R beyond P; will converge to F beyond *p*. And the contrary.

77. 6. In convex lenſes, rays diverging from R which is nearer than P; will diverge from F, on the ſame ſide with R.

78. 7. In convex lenſes, rays converging to F, will by refraction converge to R, nearer than P or *f*.

79. 8. In concave lenſes, rays converging to R, beyond P; will diverge from F, beyond *p*, and the contrary.

80. 9. In concave lenſes, rays converging to F,
81. nearer than *p*; will by refraction converge to R,
beyond F.

82. 10. In concave lenſes, rays diverging from R, beyond *p*; will after refraction diverge from F nearer than *p*.

83. 11. In concave lenſes, rays diverging from R, nearer than *p*, will diverge from F, nearer ſtill than *p*.

Note, I reckon a meniſcus, convex or concave, according as the center of its inner ſurface is further off, or nearer than the center of the outward ſurface.

And in all theſe caſes, the conjugate foci F *f*, Fig.
are ſo ſituated, that it will always be, 76.

RA : PA or *p*A : : RF : AF : : AF : *p*F to
or RP : AP : : RA : AF : : A*p* : *p*F. 83.
or RP : RA : : RA : RF : : PA : AF.

Any of theſe rules will ſerve to find the conjugate focus. All this appears from what has been before demonſtrated; particularly from Cor. 4 and 5, Prop. XX, where the rules for diverging rays for convex glaſſes is $\frac{dr}{d-r}$ = focal diſtance; and for concaves, $-\frac{dr}{d+r}$ = focal diſtance. And it is the ſame thing whether the rays diverge for convexes, or converge for concaves; all the quantities are the ſame in both, and have the ſame ſituation. Therefore if a convex and a concave have the ſame principal focal diſtance; and if a ray diverge from any focus in the convex, and another ray converge to the ſame focus, thro' the concave (put in its place); then will both have the ſame conjugate foci. And in their motions, they will deſcribe ſeparate parts of the ſame lines.

The properties of lenſes, or refracting ſurfaces, demonſtrated in the foregoing propoſitions, hold good in regard to the foci, if the mediums be changed; provided you make the rays of light go contrary ways thro' the foci, to what they do in theſe propoſitions. That is, if you ſuppoſe the rays going out of a denſe medium into a rare one, or to be refracted out of a denſe medium thro' lenſes of air, or ſome rare medium; you muſt ſuppoſe the rays to converge inſtead of diverging, and to diverge inſtead of converging; and then the focus will be found by the foregoing Rules.

PROP.

PROP. XXVI. *Prob.*

To find the principal focus of any lens.

1 *Way.*

Hold a convex lens perp. againſt the rays of the ſun, hold a white paper behind it, to receive the refracted light, which will make a round white ſpot upon the paper. Move the paper back and forward till this ſpot, which is the ſun's image, be the leaſt poſſible; there fix it. Then the diſtance of the paper from the lens, is the focal diſtance required, or the burning point.

2 *Way.*

Cover one ſide of a convex lens with paper, having ſeveral holes made in it with a pin; and placing the glaſs directly againſt the ſun beams; the light paſſing thro' the holes will make ſo many white ſpots upon a paper held behind it. Move the paper back and forward till all theſe ſpots coincide in one; there is the focus. And the diſtance of the paper from the glaſs being meaſured, gives the focal diſtance.

3 *Way.*

Place a convex lens ſo, between any object (as a window in the day, or a candle in the night) and a white wall or paper, that the image of the object may be painted diſtinctly thereon, which may be found by moving the lens back and forward. Then meaſure the diſtance of the lens from the object, and alſo from the image. Multiply them together, and divide their product by their ſum, the quotient will give the focal diſtance, by Cor. 1. Prop. XXII. Or (Cor. 2.) divide the ſquare of the diſtance of the obſerved focus, by the diſtance of the object from this focus, gives the exceſs above Fig. the principal focal diſtance. If the lens be in the midway between the object and image; then half the diſtance of the lens from either of them, is the focal diſtance.

4 *Way.*

Cover one ſide of a concave lens with paper, having two holes made with a pin, diametrically oppoſite to one another. The lens being expoſed to the ſun beams, and a paper held behind it; the light paſſing thro' the lens will make two white ſpots upon the paper. Move the paper back and forward, till the diſtance of the two ſpots on the paper, be twice the diſtance of the two holes in the cover; then the diſtance of the white paper from the lens is the focal diſtance.

PROP. XXVII. *Prob.*

To find the vertex or center of a lens.

1 *Way.*

Hold the lens at a proper diſtance from the eye, and obſerve the two reflected images of a candle, made from the two ſurfaces. Move the lens till the two images coincide, and that point is the vertex. And if this be in the middle of its ſurface, the lens is truly centered, otherwiſe not.

2 *Way.*

Draw two black lines AB, KL, croſs one another upon a white wall, or rather on paper. Take the lens CDEF, and holding it directly between your eye and the two lines, and parallel to the wall. Move it to the right and left, and you'll ſee the part *gh* of the line AB, which appears thro' the glaſs, to move back and forward. Hold the 84.

Fig. 84. glaſs at laſt, ſo that *gh* may coincide with AB; then does AB paſs thro' the vertex or center of the glaſs. Repeat the ſame thing with the line KL and the part *no*; moving the glaſs up and down, till they coincide. Mark the point Z on the glaſs, where AB, KL interſect, whilſt *gh*, *no*, coincide with them, and that interſection Z is the vertex.

Note, it may be as well to fix the glaſs, and move your eye, till A*gh*B, and K*no*L, be two continued right lines, whoſe interſection gives the vertex.

PROP. XXVIII.

Wherever the rays, which come from all the points of an object, meet again in ſo many other points, to which they converge by refraction; there they will make the picture of the object upon any white body on which they fall.

85. Let PR be any object, AB a lens. The rays that diverge from any point Q fall upon the lens, and are refracted to their focus *q*, where they all meet, and make the image of Q upon a paper held there. In like manner the rays which diverge from P, are refracted to their focus *p*, and there give the image of P; and the rays diverging from R, are refracted to *r*, and make the image of R. And thus, from all the intermediate points of the object, the rays proceeding to the lens, are refracted to as many correſpondent points in the focus; and will all together make there the image *pqr* of the object PQR, in its proper ſhape and colours. But then the picture is inverted or turned upſide down, and the right ſide to the left.

Cor. 1. *Rays flowing from all the points of an object, as* PR, *paint the image thereof pr, in the real focus q, and may be ſeen there upon a white paper; or without the paper it may be ſeen there hanging in the*

the air, by an eye at O, placed at a proper diſtance, Fig.
as 6 or 8 inches, &c. behind the focus. 85.

Cor. 2. *An object ſeen by refraction, appears in that place, from whence the rays after their laſt refraction diverge to the ſpectator's eye.*

Cor. 3. *Rays flowing from any point of an object, muſt be ſo refracted, as all to meet again in ſome other point, to give the image of that point.*

For if they ſhould fall upon different points, it would breed nothing but confuſion, for different points cannot repreſent that one point from which the rays come. All the rays flowing from the ſeveral points, muſt go to as many correſpondent points, to make the compleat image of the body. And this will appear on a paper held there; or if there be no paper, the rays will diverge from that image, as from the body at firſt; and therefore may be ſeen by an eye placed behind it, for the ſame reaſon that a body is viſible by the rays flowing from it at firſt.

PROP. XXIX.

The image of a right line immerſed in water, or any fluid, is concave towards the eye; and is ſituated in a plane paſſing thro' the eye.

Let the line AB in the fluid be parallel to the ſurface RS, and the eye at E directly over it. Then (by Cor. 6. Prop. IX.) the focus of the point C is at G, ſo that $DG = \frac{3}{4}$ DC. But (by the ſame) as the eye moves towards R or S, G will ſeem to riſe; or which is the ſame thing, as G moves further off to F or H, it will ſeem to riſe up, till it get to the ſurface RS; therefore RF and SH continually grow leſs and leſs. That is, the focus of A and B are nearer the ſurface, than G or the 86.

Fig. focus of C; and conſequently the image FGH is
86. concave towards RS.

If AB is not parallel to the ſurface, yet as one part is raiſed as much as the other is depreſſed, the image will ſtill be concave, and that if the eye be moved out of the perpendicular ED. And a plain that at firſt paſſed thro' EFABHG, will alſo paſs thro' EFGH, when the eye is moved laterally.

Cor. 1. *If the eye at* E *was in a denſe medium, and the line* AB *in a rare one. The points* A *and* B, *at a diſtance from* C, *would ſeem to deſcend; and the image of* AB *would be convex towards the ſurface* RS.

For here G would be the object and C the image, and if FGH be reduced to a ſtrait line, ACB will be reduced to a curve convex towards G. Hence therefore

Cor. 2. *A right line ſeen thro' any medium terminated by parallel planes, will appear a right line.*

For the two refractions correct one another, having contrary effects.

Cor. 3. *A plane ſurface immerged deep in water, or any fluid, will appear concave to the eye.*

Cor. 4. *The image of a ſhort right line immerged in water, does not ſenſibly differ from a right line.*

PROP. XXX.

The image of a right line made by refraction thro' a lens, is concave towards the lens.

87. Let AB be a lens, PR a right line directly expoſed to the lens. Then (by Prop. XXIV.) if P, Q, R, were equally diſtant from the lens, r

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Fig. focus of C; and conſequently the image FGH is
86. concave towards RS.

If AB is not parallel to the ſurface, yet as one part is raiſed as much as the other is depreſſed, the image will ſtill be concave, and that if the eye be moved out of the perpendicular ED. And a plain that at firſt paſſed thro' EFABHG, will alſo paſs thro' EFGH, when the eye is moved laterally.

Cor. 1. *If the eye at* E *was in a denſe medium, and the line* AB *in a rare one. The points* A *and* B, *at a diſtance from* C, *would ſeem to deſcend; and the image of* AB *would be convex towards the ſurface* RS.

For here G would be the object and C the image, and if FGH be reduced to a ſtrait line, ACB will be reduced to a curve convex towards G. Hence therefore

Cor. 2. *A right line ſeen thro' any medium terminated by parallel planes, will appear a right line.*

For the two refractions correct one another, having contrary effects.

Cor. 3. *A plane ſurface immerged deep in water, or any fluid, will appear concave to the eye.*

Cor. 4. *The image of a ſhort right line immerged in water, does not ſenſibly differ from a right line.*

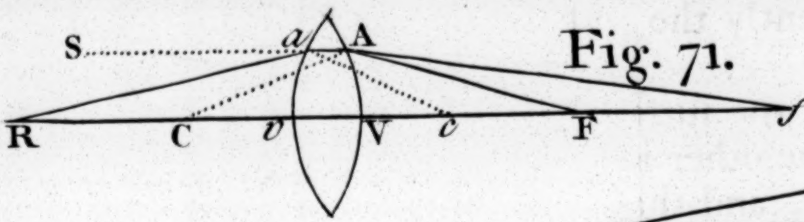
PROP. XXX.

The image of a right line made by refraction thro' a lens, is concave towards the lens.

87. Let AB be a lens, PR a right line directly expoſed to the lens. Then (by Prop. XXIV.) if P, Q, R, were equally diſtant from the lens, r

and

Fig. 71.



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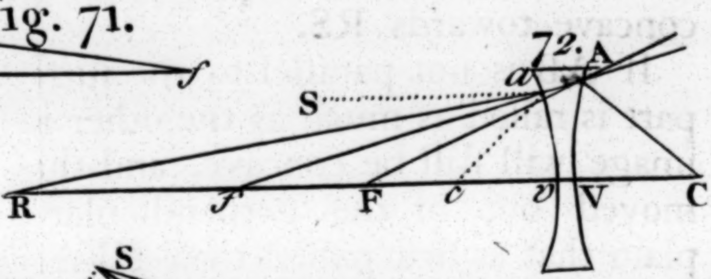

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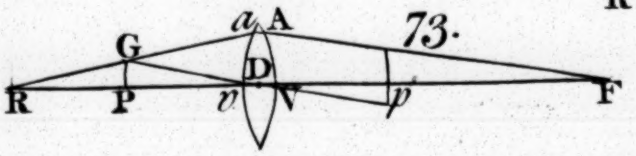

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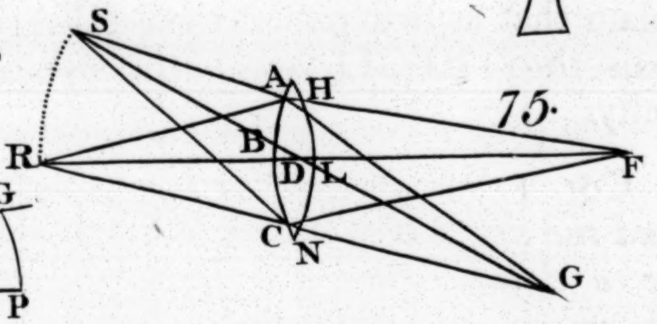

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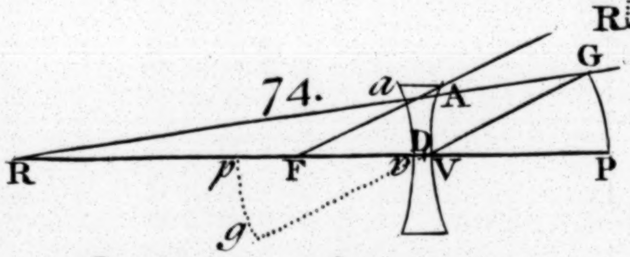

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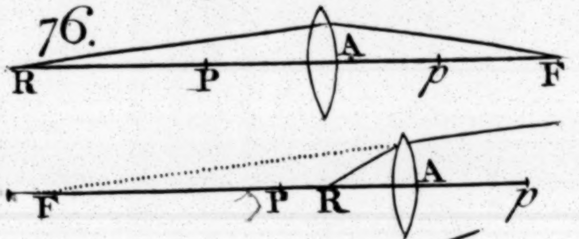

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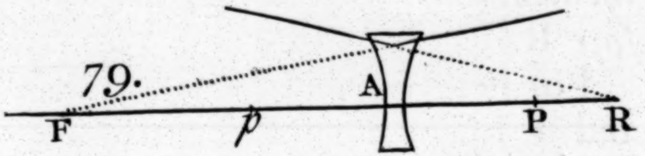

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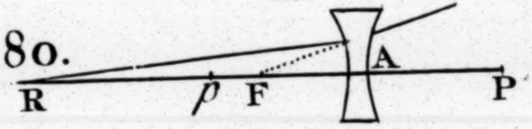

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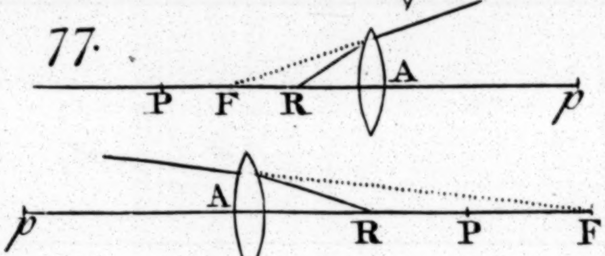

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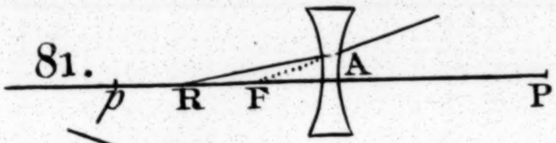

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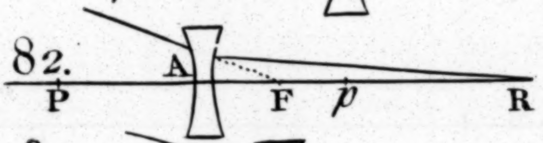

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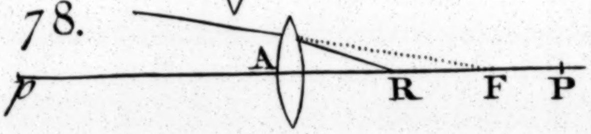

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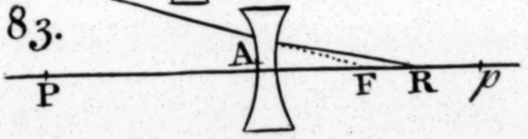

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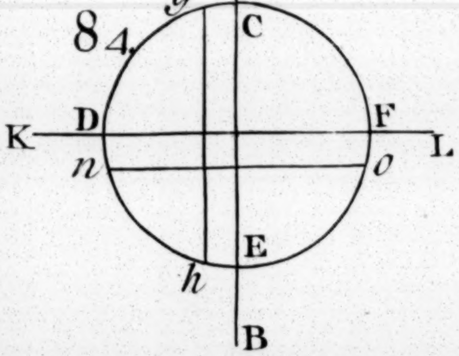

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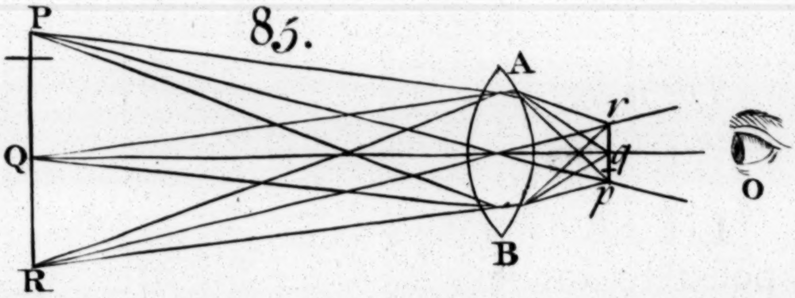

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and p would be nearer the lens than q, and therefore rqp is a curve line. But PQR being a right line, P and R are further off than Q, and that brings the focus p, and focus r nearer the lens ſtill; and conſequently rqp is more curve than before, and is concave towards the lens. And (from Cor. 2. Prop. XXIV.) the ſame holds good in a concave, and meniſcus. Fig. 87.

Cor. 1. *The image of a right line made by one refraction of a ſpherical ſurface, is alſo a little concave towards that ſurface.*

For ſuppoſing rpq made only by the firſt refraction. Then if P, Q, R be equidiſtant from the lens; (by Cor. 1. Prop. XXIV.) p, q, r would alſo be equidiſtant. But ſince P and R are ſomething further than Q; r and p will be ſomething nearer than q; or rqp will be a curve line.

Cor. 2. *The image of a ſhort right line, made by refraction thro' a lens, or thro' a ſingle ſurface, does not ſenſibly differ from a right line.*

For the deviation of the image of a very long line, from a right line, is but very little curved, and ſcarce perceptible.

Cor. 3. *The image of a circle refracted thro' a lens, to which it is concentric, will be nearly a circle alſo concentric to the lens. Or thro' a ſingle ſurface.*

For being concave to the lens, a ſmall part of it does not ſenſibly differ from a circle; whether it be thro' a lens or thro' a ſingle ſurface.

Cor. 4. *The image of a plane figure, made by refraction thro' a lens, is concave towards the lens; tho' in a ſmall figure it does not ſenſibly differ from a plane. And the ſame thro' a ſingle ſurface.*

PROP. XXXI.

The object and its image made by a lens, ſubtend equal angles at the lens.

88. For let D be the center of the lens AB, PQR
89. the object, pqr its image. Then (Cor. 2. Prop. XVII.) PDp is nearly a ſtraight line, and likewiſe RDr is alſo a ſtreight line; conſequently the angle PDR ſubtended by the object, is equal to the angle pDr ſubtended by the image, the angular point being at the center of the lens D.

Cor. 1. *Hence the object and image are equal, when they meet together.*

64. Cor. 2. *The object and image made by a ſingle refraction, ſubtend equal angles at the center of the refracting ſurface.*

For the unrefracted rays flowing from the extremes of the object, paſs thro' the center; and the object and image are contained between theſe.

PROP. XXXII.

88. *The length of an object, is to the length of the*
89. *image, made by a lens; as the diſtance of the object from the center of the lens, to the diſtance of the image from it.*

For (Prop. XXXI.) the angles PDR and pDr being equal, the triangles PDR, pDr are ſimilar; Qq being perp. to both PR and pr. Therefore $PR : pr :: PD : pD :: DQ : Dq$.

Cor. 1. *The magnitude of the object (as to its area), is to the magnitude of the image; as the ſquare of*

of the object's diſtance, is to the ſquare of the image's Fig.
diſtance, from the lens. 88.
For the areas are as the ſquares of the diameters. 89.

Cor. 2. *If* T *be the principal focus of the lens; then the diameter of the object, is to the diameter of the image;* as QT : *is to* TD.

For PR : *pr* : : DQ : D*q* : : (Cor. 1. Prop. XXI.) QT : TD.

Cor. 3. *Hence a convex magnifies an object, when it is nearer than twice the principal focal diſtance: but if further off, it leſſens it. That is, in the firſt caſe, the image is bigger, and in the ſecond, the object is bigger.*

Cor. 4. *In all caſes a concave lens diminiſhes an object, or the image is leſs than the object.*

Cor. 5. *The length of the object, and the length of its image made by a ſingle refraction; are as their diſtances from the center of the refracting ſurface.*

For (Cor. 2. Prop. XXXI.) they make equal angles at that center.

Cor. 6. *A plain glaſs, eſpecially if it be thick, magnifies a body.* 93.

For CD, EF being parallel, AC is parallel to EO, and BD to FO. Therefore AB, thro' the glaſs CF, appears of the magnitude *ab*.

Cor. 7. *If the object be cloſe to the lens; it is very little magnified thereby: or if the eye be cloſe to the lens.*

For every part of it appears nearly, in the place it is really in.

Cor. 8. *The apparent magnitude of a body, is as the angle which the image appears under to the eye in any place.*

PROP. XXXIII.

88. *If* T *be the principal focus of a lens; then the*
89. *diſtance of the object* PR *from the lens, is to the diſtance of the image pr; as* QT *to* TD.

For (Cor. 2. Prop XXXII.) QT : TD : : diameter of the object : diameter of the image : : (Prop. XXXII.) DQ : Dq.

Cor. *When* QT = TD, *the image is equal to the object.*

PROP. XXXIV.

88. *If the object and its image be both on the ſame*
89. *ſide of the lens; the image will be erect; if they be on different ſides, the image will be inverted.*

For when the rays keep the ſame poſition till they come at the image, then the image will have the ſame poſition as the object, and that is when they are both on one ſide of the glaſs. But when the rays croſs one another, before they come at the focus, or where the image is, and that is when they paſs thro' the glaſs; here the rays from the upper part of the object go to the lower part of the image, and the right to the left, &c. that is, the image will then be inverted. Thus P and p, alſo R and r, lie contrary ways by the convex lens; and the ſame way by the concave one.

90. Cor. 1. *When the object and image are on different ſides the lens, as the object approaches the lens the image recedes from it; or if the object recedes, the image approaches it. In general, if the object* Q *from an infinite diſtance approaches* D, *the image flies from the principal focus t to q; and when* Q *arrives at the principal focus* T, *q is infinitely diſtant.*

For (by Cor. 3. Prop. XXI.) QT and tq are reciprocally proportional. Fig.

Cor. 2. *If the object and image be both on one ſide of the lens; if the object moves towards the lens; the image alſo moves towards the lens. And if the object moves from it, the image moves from it.* 91.

For if t is the principal focus, tq is reciprocally as tQ, by Cor. 3. Prop. XXI.

Cor. 3. *In a convex lens, if the object be beyond the principal focus, its image will be on the other ſide of the glaſs inverted. But if the object is nearer than the principal focus, the image will be on the ſame ſide of the glaſs, erect.* 90.

For in the motion of Q to T, tq becomes infinite at T, and whilſt Q goes from T towards D, q comes up on the ſame ſide of the glaſs with Q, and then the image becomes erect.

Cor. 4. *In a concave lens, the object and image are always on one ſide of the lens.*

Cor. 5. *The diſtance of the image may be increaſed at pleaſure, and alſo its magnitude; by bringing the object nearer the principal focus.*

PROP. XXXV. *Prob.*

Having a lens whoſe principal focal diſtance is known; to find the diſtance of an object, that its diameter may be to the diameter of its image, in the given ratio of a to 1. 71.

This is eaſily reſolved by Cor. 1. Prop. XX. For ſince $d : f :: a : 1$; therefore $\frac{d}{a} = f = \frac{pdrR}{d \times \overline{r + R} - prR}$, and $d \times \overline{r + R} - prR = aprR$; there-

Fig. 71. therefore $d\times\overline{r+R} = aprR + prR = \overline{a+1}\times prR$, whence $d = \frac{\overline{a+1}}{r+R}\times prR$. Let t = principal focal diſtance; then (Cor. 1. Prop. XIX.) $t = \frac{prR}{r+R}$, and therefore $d = t\times\overline{a+1}$.

Cor. *In like manner, if all the quantities,* f, p, d, r, R, *except one, be given; that one may be found by reſolving the equation* $f = \frac{pdrR}{d\times\overline{r+R} - prR}$.

PROP. XXXVI.

The image made by a lens, becomes viſible, by placing the eye to receive the diverging rays going from the image, and at a proper diſtance for diſtinct viſion.

To ſee the image, one muſt behold it as one would do any other viſible object. And conſequently in order to ſee it, the eye muſt be placed in the diverging rays, ſo that they may enter the eye, and thereby make the image viſible. But it cannot be ſeen in any other ſituation, than in the diverging rays; for if theſe miſs the eye there can be no viſion. For as all the points of any viſible body are radiant points, ſo all the points of an image are radiant points, each in its proper pencil of rays iſſuing therefrom. All the difference is, the image can only be ſeen, when the eye is placed in thoſe pencils; whereas a luminous body, that ſend rays every way, may be ſeen in any ſituation.

Cor. 1. *The image made in the air by the lens, may not only be ſeen by the eye placed in the diverging rays; but it may be ſeen upon a paper, placed in the focus of the glaſs.*

Cor. 2. *If* R *be an object, F the focus; then to an eye at* O, *beyond the glaſs; the object* R *ſhall appear at* F; *and therefore it is magnified.* Fig. 92.

For an object appears to be ſituated in the place, from which the rays come laſt to the eye, which is at F.

Cor. 3. *Concave glaſſes, make objects appear nearer, ſmaller and fainter than they are.* 89.

For by the ſpreading of the rays, by the refraction of a concave, ſome of them miſs the eye; which would fall upon it if the glaſs was not there.

Cor. 4. *In a convex lens if the eye be placed between the glaſs and the real image; the image will appear erect, but confuſed, and nearer the eye.* 88.

For the rays do not croſs one another before they come to the eye. As if the eye was between D and *q*, there is no croſſing but at *q*, and the eye takes them before they come there. It appears nearer the eye, becauſe it is confuſed; for experience has taught us, that objects very near the eye, always appear confuſed; and therefore thro' cuſtom we judge theſe objects always to be near the eye, which appear confuſed.

Cor. 5. *Likewiſe if the eye be placed either between* D *and q, or beyond q, and you look with both eyes; the object will appear double.*

For in ſuch a poſition, the axis of the eyes cannot both of them be directed to the ſame point of the image, to make it appear ſingle.

Cor. 6. *If a convex lens be fixt in the window ſhut, and the room made dark, the pictures of external objects will be painted upon a paper held in the focus of the glaſs.*

For by Cor. 2. the images will be found there, and if the objects are very luminous, and the

Fig. room dark, they will appear bright, and in their proper colours, but inverted.

PROP. XXXVII.

94. *If an object* PR *be placed in the principal focus* Q *of a lens* AB; *its apparent magnitude at any place whatever as* O, *beyond the lens, will be invariably the ſame; and equal to the apparent magnitude when ſeen from* D, *the center of the lens, with the naked eye.*

For (by Cor. 10. Prop. XX.) all the rays of the pencil PAD, will emerge parallel to one another, and to the axis PD; and therefore the angle DOA will be equal to QDP; but (Cor. 8. Prop. XXXII.) DOA is the apparent magnitude of QP, ſeen by refraction at O, and QDP is the magnitude as ſeen from D. Therefore $\angle QDP = \angle O = \angle L = \angle I$, (which meaſures the apparent magnitude of PQ,) whether the eye be placed at O, L, or I, &c.

Cor. 1. *Hence the apparent magnitude of a body placed in the principal focus will always continue the ſame, however the eye is moved backward or forward from the lens.*

Cor. 2. *The nearer the eye is to the lens, the more of the object appears; and the further off, the leſs of it is ſeen.*

For the eye at O can ſee no more of the object than P; but at L, ſome parts above P may be ſeen by rays coming from thence thro' A to L. But if the eye be removed back from O, the ray AO will then miſs the lens, and the point P will not be viſible.

Cor. 3. *If the object be nearer than the principal focus, its apparent magnitude grows leſs in going from the*

the glaſs. If the object be further, the apparent magnitude increaſes, in going from it. Fig.

When the object is nearer than the principal focus, as at R (fig. 92.); then the focus F, and image there, will be on the ſame ſide of the glaſs; therefore the eye at O going further from the glaſs, goes alſo further from the image at F, and upon that account, it will appear leſs and leſs, but by ſlow degrees.

But when the object is beyond the principal focus as at Q (fig. 90), then the focus *q* will be on the contrary ſide of the glaſs; that is, it will be on the ſame ſide with the eye. Then let the eye be placed beyond *q* to view the image there; and as it approaches *q*, the apparent magnitude of the image will grow bigger and bigger (being as the angle it appears under), and at *q*, it becomes infinite, and infinitely confuſed, and begins to be inverted again. Then proceeding ſtill from *q* to D it decreaſes again from infinity to a given magnitude. So that on either ſide of *q*, its apparent magnitude will be as the diſtance from *q*, therefore it increaſes going from D till it be infinite at *q*.

SCHOLIUM.

There are other phœnomena of glaſs lenſes, reſolvable the ſame way, ſuch as the following.

1. If the eye be fixt in the principal focus, the apparent magnitude of an object will be invariably the ſame, wherever the object is placed beyond the glaſs.

2. If the eye is fixed at a leſs diſtance than the principal focus; the apparent magnitude of an object is diminiſhed, as it is further removed from the glaſs, but by ſlow degrees.

3. If the eye be fixed at a greater diſtance than the principal focus; the apparent magnitude of an object is increaſed, as it is removed from the glaſs; till

Fig. till it comes to the conjugate focus, in reſpect to the eye; and then it becomes infinite, and infinitely confuſed, and begins to be inverted; and going further off, it is diminiſhed again.

4. If the eye and object be fixt, and a convex lens be moved from the object to the eye; the apparent magnitude of the object increaſes to the middle, and then decreaſes to the eye. When it comes into ſuch a poſition, that the eye and object are conjugate foci, the object is infinitely confuſed.

5. If the eye and object be fixt, and a concave lens be moved from either of them to the other; the apparent magnitude of the object will decreaſe to the middle, and then increaſe again.

The apparent diſtance is reciprocally as the apparent magnitude; and in general we judge the apparent diſtance of an object to be the very ſame, as we ſuppoſe the diſtance of a real object to be, from which the rays come to our eyes with the ſame degree of confuſion.

The apparent magnitude, as has been obſerved before, is in all caſes, reciprocally as the diſtance of the eye from the image, or directly as the angle it ſubtends; and that whether the rays diverge or converge, or whether the image be before or behind the eye. But the image hardly ever appears diſtinct in any of theſe caſes.

PROP. XXXVIII.

88. *The brightneſs of the image pr, formed by the lens* AB *from the luminous object* PR; *will be as* $\frac{AB^2}{Dq^2}$, *the area of the lens directly, and the ſquare of the diſtance of the image reciprocally.*

For the brightneſs of the object being given, the quantity of light flowing from it, and falling on the lens, and tranſmitted to the image, will be directly as the magnitude of the object, and that of the lens, and reciprocally as the ſquare of the diſtance. Therefore the light falling on the picture rp, is as $\frac{PR^2 \times AB^2}{QD^2}$. But the denſity of the rays, or the brightneſs of the image, is as the quantity of light directly, and the area reciprocally, that is, as $\frac{PR^2 \times AB^2}{QD^2 \times rp^2}$. But by ſimilar triangles (Prop. XXXI) $\frac{PR^2}{pr^2} = \frac{QD^2}{qD^2}$; therefore the brightneſs is as $\frac{AB^2 \times QD^2}{QD^2 \times qD^2} = \frac{AB^2}{qD^2}$. Fig. 88.

Cor. 1. *If the brightneſs of the object be increaſed in any proportion, which call* B; *then the brightneſs of the image will be as* $\frac{AB^2}{Dq^2}$B.

Cor. 2. *In very remote objects, if f be the principal focal diſtance, then the brightneſs of the image will be as* $\frac{AB^2}{ff}$ B.

For then $Dq = f$.

Cor. 3. *The brightneſs of the image increaſes with the area of the glaſs; but the diſtinctneſs of it decreaſes.*

For it is only the rays very near the vertex, that are refracted to a ſingle point, in the picture; thoſe that are farther off deviate more and more, and ſo make the picture confuſed. All this is plain from Prop. X. and XI, where you find different proportions for the focus, when the arch is large and when ſmall, in a ſingle refraction; and the refraction at two ſurfaces increaſes the error.

PROP.

PROP. XXXIX.

95. *If a convex lens* AB *be held directly to the rays of the ſun; and a combuſtible body be held in the principal focus* P, *the heat of the rays will ſet it on fire.*

For all the rays of the ſun which fall upon the lens, are refracted into the focus, making there the image of the ſun. And all theſe rays being brought into ſo ſmall a ſpace, muſt have an intenſe heat; and the more ſo, the more rays are brought thither, or the broader the lens is. And by increaſing the breadth of the lens, the denſity of the rays, and the heat thereof may be made ſo great, as to burn or melt all ſorts of bodies placed there; as is evident by experiments.

Cor. 1. *The heat in the focus of the lens, is to the ſun's direct heat; as the area of the glaſs, to the area of the image in the focus.*

For the denſity or heat is reciprocally as the ſpace it is contained in.

Cor. 2. *The degrees of heat in the foci of different lenſes; are as the ſquares of the diameters directly, and the ſquares of the focal diſtances reciprocally.*

For the heat is as the brightneſs of the ſun's image, that is (by Prop. XXXVIII.) as $\frac{AB^2}{DP^2}$.

SCHOLIUM.

Upon account of the ſun's rays meeting in that place, and burning bodies therein, it is called the *focus* or *burning point*; and the lens itſelf a *burning glaſs*.

When the ſun's rays cannot be collected into a focus ſmall enough to burn briſkly, as in large glaſſes, or thoſe of a long focus; they may be made to paſs thro' a ſmaller lens, held in the converging rays, which will contract them, and bring them into a leſs ſpace in the focus. Fig. 95.

PROP. XL.

If AB *be a plane convex lens, expoſed with the plane ſide to the parallel homogeneous rays* NA, CV. *And if* P *be the principal focus, or that of rays falling near* V. *and* F *the focus of the ray* AF. *m*, *n* *the ſines of incidence and refraction out of a rare into a denſe medium*, $x = \mathrm{DV}$, *the verſed ſine of the arch* AV. *Then the aberration* $\mathrm{PF} = \frac{mm}{n \times \overline{m - n}}\, x$. 96.

From the center C draw CN, CM perp. to the rays NA, MP; and put $\mathrm{CV} = a$, $\mathrm{DV} = x$, $\mathrm{CP} = z$. Then $\mathrm{AD}^2 = 2ax - xx$, $\mathrm{DP}^2 = \overline{z - a + x}^2 = zz + 2xz - 2az + xx - 2ax + aa$, and $\mathrm{AP}^2 = (\mathrm{AD}^2 + \mathrm{DP}^2 =)\ zz + 2xz - 2az + aa$. By ſimilar triangles, $\mathrm{AP}^2\ (zz + 2xz - 2az + aa.) : \mathrm{CP}^2\ (zz) :: \mathrm{AD}^2$ or $\mathrm{CN}^2\ (nn) : \mathrm{CM}^2\ (mm)$, whence $nnzz = mmzz + 2mmxz - 2mmaz + mmaa$, and by reduction, $z = \frac{mma - mmx}{mm - nn} + \frac{m\sqrt{nnaa - 2mmax + mmxx}}{mm - nn}$, and putting $dd = mm - nn$, and extracting the root, $z = \frac{mm}{dd} \times \overline{a - x} + \overline{\frac{m}{dd} \times na - \frac{mm}{n}\, x - \frac{ddmmxx}{2n^3a}}$ &c. $= \frac{mma + mna}{dd} - \frac{mmn + m^3}{ndd}\, x - \frac{m^3}{2n^3a}\, xx$ &c. = (becauſe $dd =$

Fig. 96. $\overline{m+n}\times\overline{m-n})\frac{ma}{m-n}-\frac{mm}{n\times\overline{m-n}}x-\frac{m^3}{2n^3a}xx$ &c. $=$ CP. But when $x=0$, then CF $=\frac{ma}{m-n}$. Therefore CF $-$ CP or PF $=\frac{mm}{n.\overline{m-n}}x+\frac{m^3}{2n^3a}xx$ &c. $=\frac{mm}{n.\overline{m-n}}x$, nearly. Here the rays being perp. to the plain ſurface AB, ſuffer no refraction; and therefore it is the ſame thing, as if the rays were only refracted at the ſpherical ſurface AV.

Cor. 1. *Putting* AD $=y$, *the aberration by the figure of the lens, that is,* PF $=\frac{mmyy}{2an.\overline{m-n}}$.

For $yy=2ax-xx=2ax$ nearly, and $x=\frac{yy}{2a}$.

Cor. 2. *The aberration by the ſphericalneſs of the figure, is as the verſed ſine, or as the ſquare of the cord, of half the arch* AV.

Cor. 3. *The aberration in length* PF, *of a plano convex lens, is* $4\frac{1}{2}$ *times its thickneſs, nearly.*

For $m=3$, $n=2$, and $\frac{mm}{n.\overline{m-n}}x=\frac{9}{2}x=4\frac{1}{2}x$.

Cor. 4. *The lateral aberration* FG *correſponding to* PF, *is* $=\frac{mm}{nn}\times\frac{y^3}{2aa}$.

For CF $=\frac{ma}{m-n}$, and VF $=\frac{ma}{m-n}-a=\frac{na}{m-n}$. And by ſimilar triangles PD or FV $\left(\frac{na}{m-n}\right)$: AD (y) : : PF $\left(\frac{mm}{2an.\overline{m-n}}yy\right)$; FG $=\frac{mm}{nn}\times\frac{y^3}{2aa}$.

Cor. 5. *The aberration* PF $= \frac{mm}{\overline{m-n}^2} \times \frac{AD^2}{2VF}$, Fig. 96. *and the correſpondent lateral aberration* FG $= \frac{mm}{\overline{m-n}^2} \times \frac{AD^3}{2VF^2}$.

For (Prop. XIII.) the rays going into a rarer medium, $n : m :: VF : CF$. And $n : \overline{m-n} :: VF : CV$; whence $n \times CV$ (or na) $= \overline{m-n} \times VF$. Therefore PF $= \frac{mm \times yy}{2an \times \overline{m-n}} = \frac{mm \times yy}{\overline{m-n} \times 2VF \times \overline{m-n}}$. And FG $= \frac{mm}{nn} \times \frac{y^3}{2aa} = \frac{mm \times y^3}{2 \times \overline{m-n}^2 \times VF^2}$.

Cor. 6. *In a double convex lens, of the ſame focal diſtance; the aberrations will be nearly the ſame as in the plano convex.*

For the rays AP, VF will be nearly of the ſame length as before; and therefore their difference, or the aberration will be the ſame.

Cor. 7. *If the rays don't come parallel, but diverging, the aberration will be ſomething greater; and converging ſomething leſs; but may be taken for the ſame.*

Cor. 8. *If parallel rays (or any others nearly) fall upon the plane ſide of a plano convex lens, the diameter of the leaſt circle of aberration, is* $\frac{1}{4}$ *the aberration made at the principal focus; or* $2QI = \frac{1}{2} FG = \frac{mm}{nn} \times \frac{y^3}{4aa} = \frac{mm}{\overline{m-n}^2} \times \frac{AD^3}{4VF^2}$.

For this is the leaſt, when $VE = \frac{1}{2} VB$, and EV cuts AG in Q. And is demonſtrated exactly the ſame way as Prop. XXIV. B. II. And hence

Cor. 9. *The circle whoſe diameter is* 2QI, *contains all the rays refracted by the lens, between* A *and* B.

Fig. For EQ cuts AP in Q, at the greateſt diſtance
96. from the axis VF.

Cor. 10. *Hence if the principal focal diſtance is given; the lateral aberrations are as the cubes of the breadths of the lens. And if the breadth of the lens be given, the lateral aberration is reciprocally as the ſquare of the focal diſtance.*

Cor. 11. *If the focal diſtances and apertures of a reflecting concave, and a convex lens, be the ſame. The diameter of the circle of aberration cauſed by their figures, will be above 30 times leſs in the reflector than in the refractor.*

For (by Cor. 1. Prop. XXIV. B. II. and Cor. 8, of this); theſe diameters are $\frac{B^3}{4RR}$, (where f the focal diſtance $= \frac{1}{2}R$), and $\frac{mm}{nn} \times \frac{y^3}{4aa}$ (where $f = 2a$), and $B = y$. And theſe are as $\frac{B^3}{16ff}$ and $\frac{mm}{nn} \times \frac{y^3}{ff}$; that is, as $\frac{1}{16}$ to $\frac{mm}{nn}$ or as 1 to $\frac{16mm}{nn}$, or as 1 to $\frac{16 \times 9}{4}$ or 36.

PROP. XLI.

97. *Let the ſine of incidence, going out of a denſe medium be n, the ſines of refraction, (into a rare medium) of the leaſt, the mean, and the moſt refrangible rays be p, m, q. Then if a plano convex lens be expoſed with the plain ſide to the ſun; the diameter of the circle of aberration* IK *(by the different refrangibility), is to the diameter of the lens* AB; *as $q - p$, to $q + p - 2n$.*

Becauſe all the rays fall perp. on AB, they are only refracted by the ſurface AVB. Draw the radius CAL, and the axis CVF, and RAG, SBH Fig. 97. parallel to CV. And let AI*f* and BK*f* be the leaſt refrangible rays, and AFK, BFI, the moſt efrangible. Thro' I, K draw LH which will be perp. to C*f*. Now ſince the arch AV is ſuppoſed to be very ſmall, the angle ACV is very ſmall, and conſequently the angles LAG, LAI, LAK are likewiſe very ſmall; and conſequently the lines LG, LI, LK are nearly as the ſines of theſe angles; therefore if LG $= n$, then will LI $= p$, LK $= q$: For LAG or RAC is the angle of incidence. Hence GI = (LI — LG =) $p-n$, IK = (LK — LI =) $q-p$. And IO = ½IK $=\frac{q-p}{2}$. Therefore GO or AD = (GI + IO =) $p-n+\frac{q-p}{2}=\frac{p+q-2n}{2}$; and 2AD $=p+q-2n$. Therefore 2AD or AB : IK : : $p+q-2n : q-p$. And all the rays between A and B, paſs thro' IK.

Cor. 1. *Diameter of the circle of aberration* IK : *diameter of the lens* AB : : $\frac{q-p}{2} : m-n$.

For $p+q=2m$, and IK : AB :: $q-p : 2m-2n : \frac{q-p}{2} : m-n$.

Cor. 2. *The aberration* F*f* : *focal diſtance* *f*D : : $q-p : m-n$.

For by ſimilar triangles, *f*O : OI : : *f*D : AD; and 2*f*O or *f*F : 2OI or IK : : *f*D : AD or ½AB. And *f*F : *f*D : : IK : ½AB : : $\frac{q-p}{2} : \frac{m-n}{2}$: : $q-p : m-n$.

Cor. 3. *In a plano convex glaſs the diameter of the circle of aberrations, or that which cuts all the rays, is the 55th part of the diameter of the glaſs. And the*

Fig. 97. *the aberration in length, is the 27th or 28th part of the focal diſtance.*

For in glaſs $n = 50$, $p = 77$, $q = 78$. And AB : IK : : $m - n$ $(27\frac{1}{2})$: $\frac{q-p}{2}$ $(\frac{1}{2})$: : 55 : 1. And $fD : Ff$: : $m - n$ $(27\frac{1}{2})$: $q - p$ (1).

Cor. 4. *The diameter of the circle of aberrations, in which all the rays are contained, that affect the ſenſe, is hardly the 200th part of the diameter of the glaſs.*

The foregoing computation takes in all the rays, but ſince they are exceeding thin toward the circumference of the circle of aberrations, and exceeding denſe near the center; not above a quarter of that makes any ſenſible impreſſion. And therefore 4 × 55 or 220 to 1 will now be the ratio of AB to KI.

Cor. 5. *In different glaſſes, the aberrations are as the diameters of the glaſſes. And the angles of aberration are as the breadth of the glaſſes directly, and the focal diſtance, inverſely.*

96. 97. Cor. 6. *Hence if* b *be the breadth of the lens,* f *the focal diſtance; the aberration by the figure of the lens, is to the aberration by the different refrangibility; as* $\frac{mm}{m-n} \times \frac{bb}{16ff}$ to $q - p$.

For Cor. 8. Prop. XL.) the diameter by the figure is $\frac{mm}{\overline{m-n}^{2}} \times \frac{AB^{3}}{8 \times 4VF^{2}} = \frac{mm}{\overline{m-n}^{2}} \times \frac{b^{3}}{32ff}$. And the diameter by the aberration (by Cor. 1. of this) is $\frac{q-p}{2 \times \overline{m-n}} \times b$. Divide both by $\frac{b}{2 \times \overline{m-n}}$, and you have the ratio as above.

Cor. 7. *In the glaſſes of teleſcopes, the error cauſed by the figure of the glaſſes, is quite inconſiderable, and may be reckoned as nothing, in reſpect of the error by the different refrangibility.* Fig. 96. 97.

For in glaſs $n = 50$, $p = 77$, $m = 77\frac{1}{2}$, $q = 78$, and the error by the figure, is to the error by the different refrangibility, as $\frac{13.65bb}{ff}$ to 1. For $q - p = 1$, and

$$\frac{mm}{m-n} \times \frac{bb}{16ff} = \frac{77\frac{1}{2} \times 77\frac{1}{2}}{27\frac{1}{2}} \times \frac{bb}{16ff} = \frac{31 \times 77\frac{1}{2}}{11} \times \frac{bb}{16ff} = \frac{31 \times 155}{11 \times 32} \times \frac{bb}{ff} = 13.65 \times \frac{bb}{ff}.$$

Ex. 1.

Suppoſe $b = 4$ inches, $f = 100$ feet $= 1200$ inches, then $\frac{13.65bb}{ff} = \frac{13.65 \times 16}{1440000} = \frac{218.4}{1440000}$, and the error by the figure is to the error by refraction, as 218.4 to 1440000, or as 1 to 6593. And if the rays that affect the ſenſe be only a quarter of this, (as in Cor. 4.), the ratio becomes as 1 to 1648.

Ex. 2.

Suppoſe $b = 2$ inches, $f = 240$ inches; and $\frac{13.65bb}{ff} = \frac{13.65 \times 4}{240 \times 240} = \frac{13.65}{120 \times 120} = \frac{13.65}{14400} = \frac{1}{1055}$; and theſe errors as 1055 to 1. And if we reckon only the mean rays; it will be no leſs than 263 to 1; a trifling quantity for the figure of the glaſs.

From hence it appears, that the leſſer the focal diſtance of the glaſſes are, the more conſiderable is the error proceeding from the figure of the glaſs. And therefore in microſcopes, the aberration ariſing

Fig. 96. 97. from the ſpherical figure of the glaſſes, may have a ſenſible effect; by reaſon of the ſmallneſs of the ſpheres, and the large apertures required therein.

Cor. 8. *If the breadth of a glaſs lens be* $\frac{2}{3}$ *the diameter of the ſphere; the aberration cauſed by the figure, is nearly equal to the aberration cauſed by the different refrangibility of the rays.*

For by Cor. 6. the aberration by the figure is to the aberration by the different refrangibility; as $\frac{mm}{m-n} \times \frac{bb}{16ff}$ to $q-p$. Put them equal, then $\frac{mm}{m-n} \times \frac{bb}{16ff} = q-p$, or in numbers, $\frac{9}{1} \times \frac{bb}{16ff} = 1$. And $9bb = 16ff$. And extracting the root, $3b = 4f = 4 \times$ radius in a double convex $= 2$ diameters, and $b = \frac{2}{3}$ the diameter.

Cor. 9. *The aberration in any lens by the different refrangibility, will be nearly the ſame, as in a plano convex of the ſame focus.*

For in a double convex of equal radii, the refraction at each ſurface will be but half ſo much as at the convex ſurface of the plano convex, which has double the curvature.

Scholium.

Altho' glaſſes formed into the figures of the conic ſections or any other curves, could refract the rays accurately to one point. Yet ſince no method has been found out to give them theſe figures, by grinding them into their proper forms; and a glaſs may be very truly ground into the form of a ſphere. Therefore Opticians are obliged to make uſe of ſpherical glaſſes upon all occaſions. And if we had glaſſes in the form of any curve, we chuſe, yet none

one of them can, in the leaſt, alter the different Fig.
efrangibility of the rays. Neither can any curve 96.
efract the rays iſſuing from the collateral parts of an 97.
bject, to a ſingle point. And therefore ſince all he errors of glaſſes are to be attributed ſolely to he different refrangibility of the rays. Therefore ſpherical ſurfaces may be of the ſame uſe as the beſt curves; and being ſo eaſily made, they are more commodious for dioptrical uſes, than any other curves whatever.

BOOK IV.

The Combination of GLASSES; *and th* *Conſtruction of* TELESCOPES *and* MICROSCOPES, *and other Optical Inſtruments.*

PROB. I.

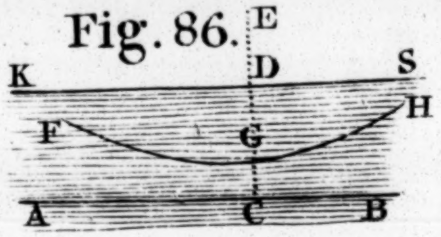
Fig. *To explain the Nature and Conſtruction of the Eye.*

AS the ſenſe of ſeeing is the nobleſt ſenſe be longing to an animal, without which h would live in perpetual darkneſs, unable to perfor the neceſſary exerciſes of life; and ſince all th has been demonſtrated of glaſſes, and will herea ter be demonſtrated of all optical machines, ten only to this ſingle end, to help and improve th ſenſe of ſeeing. And ſince the eye is the orga or inſtrument by which viſion is performed; it by its nature an optical inſtrument, and the fou-dation of all others. And therefore it cannot b amiſs to give a ſhort deſcription thereof.

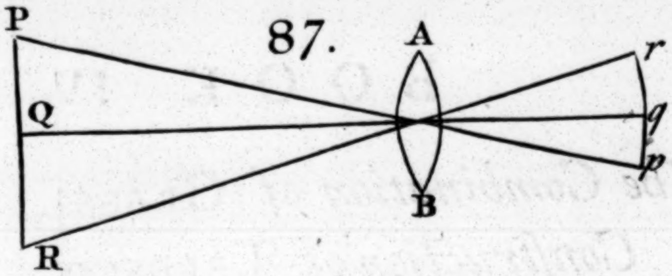
98. ABCE is the eye, its figure ſpherical; by reaſo of which, it is eaſily moved any way in its ſocke The fore part at A is more convex than the reſ It is contained in three membranes, the outermo is the *Sclerotica*; the ſecond the *Tunica choroides* the fore part of it is called the *Iris*, which conſiſt of many fibres like ſo many radii. The third o

inner-

Fig. 86.



87.



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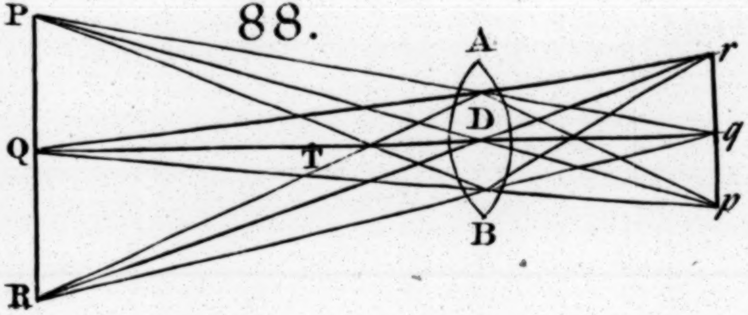

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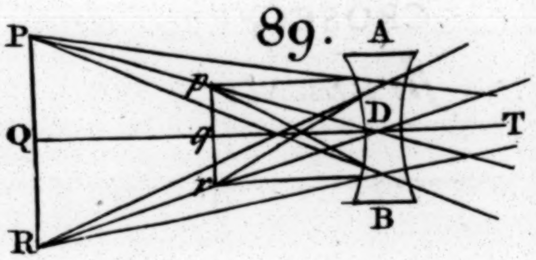

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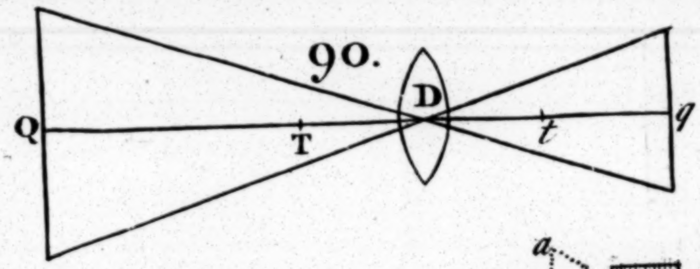

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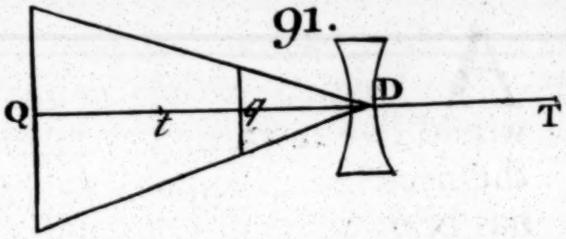

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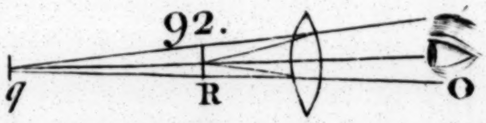

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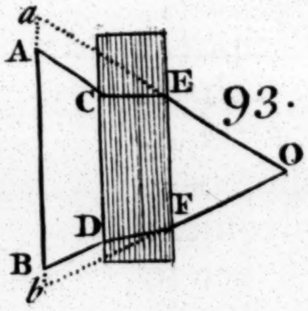

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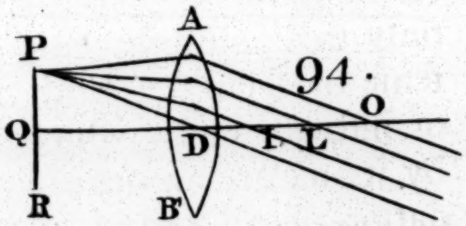

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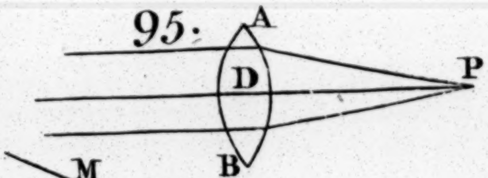

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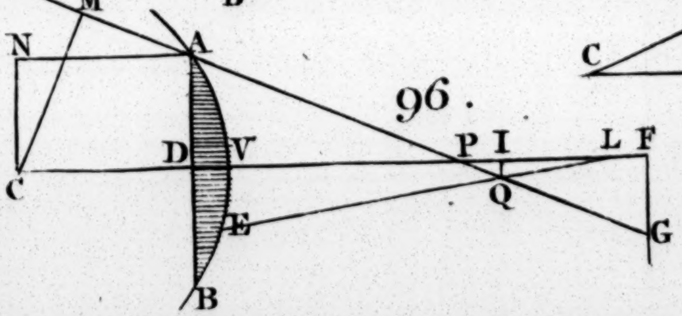

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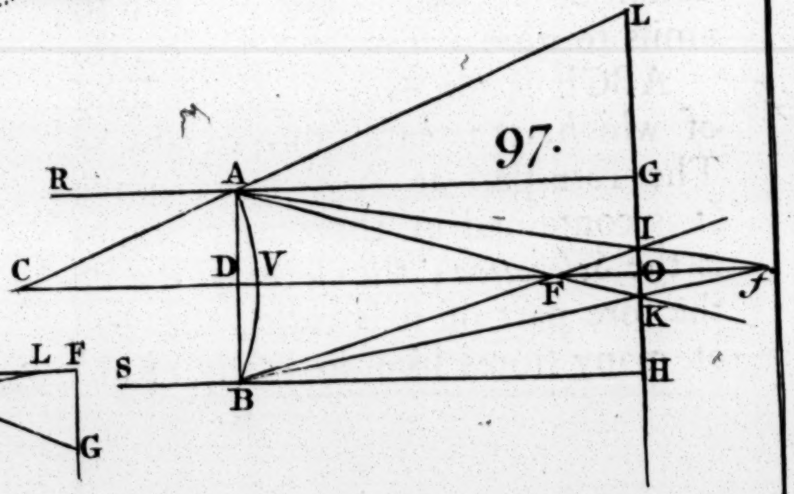

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nnermoſt is called the *Retina*, which is nothing Fig. 98. but the optic nerve, ſpread over the bottom of the eye.

In theſe are contained the three humours of the eye; the firſt is HAI, called the *Aqueous* humour, which is a thin liquor like water. The ſecond is FG the *Chriſtalline*, in the form of a lens, more convex behind towards C. Adjoining to this is the third KL, called the *Vitreous* humour.

The chriſtalline is more denſe than the vitreous, and the vitreous more denſe than the aqueous humor, and all together make a compound lens, which refracts the rays of light, iſſuing from an object PR, to the bottom of the eye, and there paints its image *pr* in the focus upon the retina, inverted.

The figure of the aqueous humour is a meniſcus, and ſo is that of the vitreous. The fore part of the ſclerotica is called the *Cornea*, and that part adjoining to it, is called the *White* of the eye. Within the cornea is a coat called the *Uvea*; in the middle of this is a hole O called the *Pupil*, to let in the rays of light. And the pupil is contracted or dilated by ſeveral muſcular fibres in the uvea or iris, according as more or leſs light falls on the eye, and that by an involuntary motion.

D is the optic nerve going from the retina to the common ſenſorium in the brain, where theſe images are perceived. This is not in the middle of the eye, but lies nearer the ſide E, which is towards the noſe.

Round the edge of the chriſtalline FG, is a ring of fibres, by help of which the diſtance AC is made longer or ſhorter, in order to bring the image *pr* upon the retina, for diſtinct viſion. And perhaps theſe fibres alſo make the chriſtalline more or leſs convex at the ſame time. This ring of fibres is called the *Ligamentum ciliare*; and its back part is

Fig. 98. is black, to ſtifle the rays that are reflected upon it.

The eye is moved in the head by ſeveral muſcles inſerted in the ſclerotica, ſo that it may be quickly directed to any object.

If the image of an object does not fall upon the retina at *pr*, the viſion will be confuſed. If it fall ſhort, or nearer FG; then a concave lens that makes the rays more diverging will bring them to the retina. This is the caſe of purblind or ſhort-ſighted people, who are forced to look very near an object; or elſe they muſt uſe concave ſpectacles.

If the rays do not unite till they get beyond the retina, as in moſt old people; then a convex lens of a due focus, held between the eye and object, will make them converge, and fall upon the retina. Therefore long-ſighted people muſt uſe convex ſpectacles.

Altho' the image is painted inverted in the eye, yet we judge it erect; becauſe we always find it ſo by cuſtom and experience.

The diameters of images at the bottom of the eye, as *pr*, are proportional to the angles which the objects ſubtend at the eye, as POR; the ſame as in a lens; and are reciprocally as the diſtances of the ſame object viewed in different places.

The eye is in reality no more than a camera obſcura, for the rays of light flowing from all the points of an object, thro' the pupil of the eye, do by the refraction of the humours of the eye, paint the image thereof in the bottom of the eye. Juſt ſo it is in the camera obſcura, where all the rays refracted by a lens in the window ſhut, or paſſing thro' a ſmall hole in it, paint the image on the oppoſite wall. Some properties of the eye are theſe.

1. The eye can only ſee a very ſmall part of an object diſtinctly at once. For the collateral parts of

f an objeſt are not repreſented diſtinctly in the Fig. 98. ye; and therefore the eye is forced to turn itſelf ucceſſively to the ſeveral parts of the object it ants to view, that they may fall near the axis f the eye, where alone diſtinct viſion is performed.

2. When any point of an object is ſeen diſtinctly ith both eyes, the axes of both eyes are directed that point and meet there. And then the obct appears ſingle tho' lookt at with both eyes. or the optic nerves are ſo framed, that the coreſpondent parts in both eyes, lead to the ſame lace in the brain, and give but one ſenſation; and e image will be twice as bright with both eyes as ith one. But if the axes of both eyes are not irected to the object, that object will appear doule, as the pictures in the two eyes do not fall pon correſpondent or ſimilar parts of the retina.

3. The beſt eye can hardly diſtinguiſh a partile of matter that ſubtends at the eye, an angle eſs than half a minute. And very few can diſinguiſh it when it ſubtends a minute. If the diſance of two ſtars in the heavens be not greater han this, they will appear as one.

4. Young people can ſee nearer hand than old eople; for as men grow old their eyes grow flater; and to collect the rays of light at the retina, he object muſt be placed further off. And this is he reaſon that ſhort-ſighted people as they grow lder ſee at a greater diſtance.

5. Tho' men may ſee diſtinctly at different diſances, by altering the poſition and figure of the hriſtalline, yet they can only ſee diſtinctly within ertain limits, and nearer than that, objects appear onfuſed. But theſe limits are not the ſame in ifferent people. A good eye can ſee diſtinctly, hen the rays fall parallel upon it; and then the rincipal focus is at the bottom of the eye.

6. If

Fig. 98. is black, to ſtifle the rays that are reflected upon it.

The eye is moved in the head by ſeveral muſcles inſerted in the ſclerotica, ſo that it may be quickly directed to any object.

If the image of an object does not fall upon the retina at *pr*, the viſion will be confuſed. If it fall ſhort, or nearer FG; then a concave lens that makes the rays more diverging will bring them to the retina. This is the caſe of purblind or ſhort-ſighted people, who are forced to look very near an object; or elſe they muſt uſe concave ſpectacles.

If the rays do not unite till they get beyond the retina, as in moſt old people; then a convex lens of a due focus, held between the eye and object, will make them converge, and fall upon the retina. Therefore long-ſighted people muſt uſe convex ſpectacles.

Altho' the image is painted inverted in the eye, yet we judge it erect; becauſe we always find it ſo by cuſtom and experience.

The diameters of images at the bottom of the eye, as *pr*, are proportional to the angles which the objects ſubtend at the eye, as POR; the ſame as in a lens; and are reciprocally as the diſtances of the ſame object viewed in different places.

The eye is in reality no more than a camera obſcura, for the rays of light flowing from all the points of an object, thro' the pupil of the eye, do by the refraction of the humours of the eye, paint the image thereof in the bottom of the eye. Juſt ſo it is in the camera obſcura, where all the rays refracted by a lens in the window ſhut, or paſſing thro' a ſmall hole in it, paint the image on the oppoſite wall. Some properties of the eye are theſe.

1. The eye can only ſee a very ſmall part of an object diſtinctly at once. For the collateral parts of

Fig. 98.

f an object are not repreſented diſtinctly in the ye; and therefore the eye is forced to turn itſelf ucceſſively to the ſeveral parts of the object it ants to view, that they may fall near the axis f the eye, where alone diſtinct viſion is performed.

2. When any point of an object is ſeen diſtinctly ith both eyes, the axes of both eyes are directed o that point and meet there. And then the obect appears ſingle tho' lookt at with both eyes. or the optic nerves are ſo framed, that the coreſpondent parts in both eyes, lead to the ſame lace in the brain, and give but one ſenſation; and he image will be twice as bright with both eyes as ith one. But if the axes of both eyes are not irected to the object, that object will appear doule, as the pictures in the two eyes do not fall pon correſpondent or ſimilar parts of the retina.

3. The beſt eye can hardly diſtinguiſh a partile of matter that ſubtends at the eye, an angle eſs than half a minute. And very few can diſinguiſh it when it ſubtends a minute. If the diſance of two ſtars in the heavens be not greater han this, they will appear as one.

4. Young people can ſee nearer hand than old eople; for as men grow old their eyes grow flater; and to collect the rays of light at the retina, he object muſt be placed further off. And this is he reaſon that ſhort-ſighted people as they grow lder ſee at a greater diſtance.

5. Tho' men may ſee diſtinctly at different diſances, by altering the poſition and figure of the hriſtalline, yet they can only ſee diſtinctly within ertain limits, and nearer than that, objects appear onfuſed. But theſe limits are not the ſame in ifferent people. A good eye can ſee diſtinctly, hen the rays fall parallel upon it; and then the rincipal focus is at the bottom of the eye.

6. If

Fig. 98. 6. If a good eye views an object at the leaſt diſtance it can be ſeen diſtinctly, and then at twice that diſtance, and then at an infinite diſtance; there is about the ſame alteration made in the figure of the eye between the two laſt caſes, as there is between the two firſt.

7. If a ſhort-ſighted perſon can ſee an object diſtinctly at two diſtances, whereof one is double the other. As great an alteration muſt be made in the figure of their eyes to ſee it at an infinite diſtance, as there was made for the firſt two diſtances; or as great an alteration as is made in a good eye, to ſee diſtinctly from the greater diſtance to infinity. And therefore a ſhort-ſighted perſon can ſee diſtinctly at all diſtances with a ſingle concave glaſs, of a proper figure. And hence the cauſe of ſhort-ſightedneſs is not a want of power to change the figure of the eye; but that change is not great enough to accommodate it to all diſtances, taken from a point ſo near the eye.

8. A man can judge of a ſmall diſtance with a ſingle eye, by frequently obſerving how much variation is made in the eye to make the object diſtinct; and from this a habit of judging is acquired. But this cannot be done at great diſtances, becauſe tho' the diſtance be varied, the change in the eye becomes then inſenſible.

9. But a man can judge of greater diſtances with both eyes, than he can with one eye. For the eyes being at a diſtance from one another, as long as that diſtance has a ſenſible proportion to the diſtance of the object, one gets a habit of judging, by the poſition of the two axes of the eyes, which are always directed to that point. For different diſtances require different poſitions, of the axes, which depends on the motions of the eyes which we feel. But in very great diſtances, no judgment can be made from the motion of the

eyes

es, or their internal parts. Therefore we can Fig. ly gueſs at the diſtance from the magnitude, co- 98. ur, and the poſition of interjacent bodies.

10. Whatever light falls upon that part of the tina, where the optic nerve D, ſprings, makes impreſſion; and therefore if the picture of an bject falls thereon, it is not perceived, and ſo that bject is inviſible. This will appear by placing a nall bright object before you, and looking at it ith one eye, and moving your eye laterally to- ards the contrary ſide (towards the left, if it be e right eye), the object will diſappear, and ſeem be loſt; and moving it ſtill further, it will ap- ear again. Now this place is not at the bottom f the eye, but nearer the noſe in both of them; ſo at no rays either parallel or diverging, that come rom any object, can fall upon that place in both e eyes; ſo that any object we direct the eyes to, ill always be viſible, at leaſt to one eye. But he ſame bright object may be made to diſappear both eyes, by directing the axes of both eyes to point a little beyond the noſe, to be found by rials.

11. Dimneſs of ſight generally attend old peo- le, and this may ariſe from two cauſes, 1. by the yes growing flat, and not uniting the rays at the etina, which cauſes indiſtinctneſs of viſion; or, . by the opacity of the humours of the eye, which n time loſe their tranſparency in ſome degree; rom whence it follows that a great deal of the ight is ſtopt and loſt, that enters the eye; and very object appears faint and dim.

12. As the rays of light flowing from an ob- ect, and painting its image upon the retina, is he immediate cauſe of ſeeing. So where there is o light, there can be no viſion; conſequently without light, the eye becomes a machine utterly uſeleſs;

Fig. 98. uſeleſs; as it can give us no manner of information of the exiſtence of bodies at a diſtance from us.

PROB. II.

To find the places and poſitions of the images formed by ſeveral lenſes, or reflectors.

There may be an infinite number of caſes, of this problem, according to the number of lenſes and their diſtances from one another; but any of them may be ſolved in general thus.

Draw a right line thro' the centers of all the lenſes, for their common axis. Then having the poſition of the object, find the focus of the firſt glaſs, or the place of the firſt image. This being had, and the place of the ſecond lens; find the other focus, or the conjugate focus of the ſecond lens, or the place of the ſecond image. This being found, and the place of the third lens; find its conjugate focus, or the place of the third image, and ſo on from one to another, as long as there are any lenſes, till you get the laſt image.

The places of the images being known, their poſition as to their being erect or inverted may be found thus. Take a point in the object above the axis, from which draw a line thro' the center of the firſt glaſs, and continue it to the focus, there you will have the firſt image of that point, and if it is above the axis it is erect; but if below, 'tis inverted. From that point draw a line thro' the center of the ſecond glaſs, to the ſecond image, and that gives the ſecond image of that point, which image is erect or inverted, according as that point is above or below the axis. After the ſame manner purſue the ray thro' the third glaſs, and then thro' the fourth glaſs, and ſo on to the laſt, where if the laſt image of the ſaid point be

above the axis, the image is erect; if below, it is Fig.
inverted. If there be any reflectors, purſue the ray, after the ſame manner, to the reſpective foci. The ſhorteſt way to find the conjugate foci, of all the glaſſes, ſucceſſively, is to get the principal foci firſt; and then theſe will be had by Prop. XXI. Book III.

Ex. 1.

Let AB be a reflecting concave, PQ an object, 99.
draw the axis QD, and find the focus *q*. From P the higheſt point of the object, draw PD, and the reflected ray D*p*, then *p* is the image of P, and *pq* the image of PQ; and ſince *qp* is below the axis, the image is inverted.

Ex. 2.

Let A, B, C, be three lenſes, PQ an object. 100.
Draw QH the common axis of all the lenſes; in the axis QH find the conjugate foci Q, F, of the lens A; and F, G, the conjugate foci of the lens B; and laſtly G, H the conjugate foci of the lens C. Take the point of the object P above the axis, and thro' A the center of the firſt lens, draw PA, and continue it to L at the focus F; then L is the image of P. Again, draw LBM thro' the center B of the ſecond lens, to the focus G; then M is the image of L, or the ſecond image of P. Laſtly, draw MCN thro' the center C of the third lens, to the focus at H; and N is the image of M, or the third image of P. And becauſe HN lies below the axis, or contrary to the poſition of PQ, the laſt image is inverted.

Ex. 3.

Let A, B, C, D, be four lenſes, PQ an object. 101.
Draw QK the common axis of all the lenſes. In this axis find the conjugate foci Q, F, of the firſt lens; F and G, the conjugate foci of the ſecond

Fig. 101. lens; G and H, the conjugate foci of the third lens; laſtly, H and I the conjugate foci of the fourth lens, which is a concave. Take a point of the object P, above the axis, and draw PAL thro' the center of the firſt lens to the focus F; then L is the image of P. Draw LB to the center of the ſecond lens, cutting the perpendicular GM, at the focus G, in M; then M is the image of L, or the ſecond image of P. Here the rays, which by the glaſs A, converged to L; by the glaſs B are made to converge to M. Again, draw MCN thro' the center of the third glaſs, to cut the perp. HN at the focus H, in N; and N is the image of M, or the third image of P. Laſtly, draw ND to the center of the fourth glaſs, cutting the perp. IO, at the focus I, in O; and O is the image of N, or the fourth image of P. Here the rays diverging from N, and falling on the concave glaſs D, are made to diverge from O; juſt as the rays diverging from H, are made to diverge from I. And ſince IO is on the ſame ſide of the axis, as the object PQ, it will appear to be erect.

And thus the rays proceeding from an object, may be traced thro' any number of lenſes, to the place of the laſt image.

Cor. 1. *Hence, if part of the rays are ſtopt by the way, what remains will belong to the ſame focuſes as before; and the ſame images will be made by theſe partial pencils of rays that remain.*

For in the oblique pencils, a great many rays miſs the following glaſſes, and are loſt; and the more ſo, the more oblique they are. But that does not affect the reſt, which will ſtill go to their own focus.

Cor. 2. *Hence the proportion of the laſt image to the object is eaſily found. And that is, as* IO : PQ : : *ſo* *ſo* AF × BG × CH × DI : *to* QA × BF × CG × DH. Fig. 101.

For PQ to IO is compounded of the ratio of PQ to FL, FL to MG, MG to HN, and HN to IO; that is, of QA to AF, BF to BG, CG to CH, and DH to DI; or of QA × BF × CG × DH to AF × BG × CH × DI.

Cor. 3. *Likewiſe the apparent magnitude of the laſt*
image IO *ſeen from any point* K :
is to the apparent magnitude of the object PQ *ſeen*
from A, *with the naked eye* : :
as DI × CH × BG × AF :
to KI × DH × CG × BF.

For the apparent magnitude of IO, to the apparent magnitude of PQ, is as the angle IKO to the angle QAP. But IKO to QAP is compounded of the ratios of IKO to IDO, and IDO or HDN to HCN, and HCN or GCM to GBM, and GBM or FBL to FAL or QAP; that is, of the ratios of DI to KI, and CH to DH, and BG to CG, and AF to BF; or as DI × CH × BG × AF to KI × DH × CG × BF.

Cor. 4. *Therefore where there are but three lenſes,* A, B, C; *and the eye at* C. *The apparent magnitude of the laſt image* GM : *is to the apparent magnitude of the object* PQ *ſeen from* A : : *as* BG × AF : *to* CG × BF.

PROB. III.

To find the place of the eye, where the viſible area of the object is the greateſt, when ſeen thro' ſeveral glaſſes. 102.

This will be in the place where the axes of all the pencils meet. Suppoſe A the centre of the firſt glaſs, to be a radiant point, and that the rays iſſuing from A, are refracted ſucceſſively by all the

Fig. 102. glaſſes BC, EF, &c. Then find the focus G made by refraction at the laſt glaſs EF; and G is the place of the eye. And this is found after the manner of the laſt Prob. by finding the conjugate foci A, D, for the glaſs BC; and then the conjugate foci, D, G, for the glaſs EF, and ſo on to the laſt, which is done either by Prop. XXI. or Cor. 2. Prop. XX. of Book III.

For ſince the glaſs A is the common baſe of all the pencils of rays that come from the object; the axis of every pencil, LA, OA, NA, will paſs ſtreight thro' the center A of the glaſs, and LAC, OAD, NAB, &c. will be right lines. Therefore A is the radiant point from which all theſe axes proceed. Therefore it follows of courſe, that all theſe axes or middle rays interſect again in the ſeveral foci D, G &c. And conſequently if the eye be placed at the laſt focus G, it will receive all theſe middle rays, and beſides as many of the collateral rays of every pencil, as the breadth of the pupil can admit. Therefore the viſible area of the object will be greateſt, when the eye is in the focus G. For if the eye be ſuppoſed at any other place, the eye will take in leſs, becauſe the ſection of all the rays becomes greater; (for it is the very leaſt at D), and fewer of them will fall on the pupil.

Cor. 1. *If the laſt glaſs* EF *be a concave, the eye muſt be put cloſe to the glaſs.*

For its focus G will fall between the glaſs EF and the point D; and the rays coming from it thro' the glaſs will diverge, and the ſection of the rays, on the outſide of the glaſs, will be the leaſt neareſt the glaſs.

Cor 2. *The focus of the axes, of all the pencils, is a quite different thing from the focus of any one pencil.*

For the former has its radiant point in the centre Fig. of the firſt glaſs; and the latter, in the object itſelf. 102.

Cor. 3. *For the ſame reaſon, if* A *be a concave reflector, and* A, D, *conjugate foci to* BC; D *will be the interſection of all the axes of the ſeveral pencils, or the place of the eye.*

PROB. IV.

Having given AF, *the leaſt diſtance that a long-ſighted perſon can ſee diſtinctly; or the greateſt diſtance a ſhort-ſighted perſon can ſee diſtinctly; and* AQ *the diſtance he wants to ſee diſtinctly. To find the radius of a glaſs fit for that purpoſe.* 103. 104.

Let AC be the radius of a double convex, or double concave; take $AC = \frac{AF \times AQ}{AF - AQ}$, for the radius of a convex lens, for the long-ſighted perſon; or for the radius of the concave, for the ſhort-ſighted perſon.

For if C be the principal focus of the double convex, or double concave lens; or which is the ſame thing, CA = the radius of the ſphere (by Art. 1. Schol. Prop. XIX. B. III.) Then (Prop. XXII. ib.) $QF : QA :: AF : CA = \frac{AF \times AQ}{QF} = \frac{AF \times AQ}{AF - AQ}$. Then the rays that come from Q, will enter the eye of the long-ſighted perſon, as if they came from F, and will therefore make the object at Q appear diſtinctly (fig. 103.) and the rays proceeding from Q (fig. 104.), will enter the eye of the ſhort-ſighted perſon, as if they came from F; and will therefore make the object at Q appear diſtinctly to him, when AC the radius of concavity $= \frac{AF \times AQ}{AF - AQ}$, or $\frac{AF \times AQ}{AQ - AF}$.

Fig. 103. Cor. 1. *A long-ſighted perſon that cannot ſee diſtinctly nearer than* F; *cannot ſee diſtinctly nearer than* Q, *by help of the glaſs* BD. *But from* Q *to* C, *or further, he can ſee diſtinctly.*

For as F approaches the glaſs, Q approaches it (Cor. 2. Prop. XXXV. B. III.); but nearer than F, the object is confuſed; therefore nearer than Q, the image is confuſed. But beyond F it is diſtinct, therefore beyond Q it is diſtinct.

Cor. 2. *If the long-ſighted perſon can only ſee objects diſtinctly, which are very remote; then he muſt have a convex lens whoſe focal diſtance is* AQ, *to ſee objects diſtinctly at the diſtance* AQ.

For in this caſe the radius is $\frac{AF \times AQ}{AF} = AQ$, becauſe AF is infinitely greater than AQ.

104. Cor. 3. *A ſhort-ſighted perſon that cannot ſee diſtinctly further than* F; *cannot ſee diſtinctly further than* Q, *by help of the concave glaſs* BD. *But can ſee diſtinctly from* Q *to* F, *or nearer.*

For as F and Q both recede from the glaſs; and beyond F, the object is confuſed, ſo is the image beyond Q; but on this ſide of Q it is diſtinct.

Cor. 4. *If* AF *be the greateſt diſtance, a ſhort-ſighted perſon can ſee diſtinctly; then by help of a concave, whoſe radius or focal diſtance is* AF, *he will be able to ſee all remote objects, to infinity.*

For in this caſe AQ is infinite, and the rays from Q will be parallel; and conſequently F is the focus of theſe parallel rays, and FA the radius of the glaſs. And in all remote objects, the rays are nearly parallel.

Cor. 5. *From theſe rules a man may be fitted with the moſt proper ſpectacles, tho' he lives at a diſtance from*

from the maker; by ſending him the focal diſtances. But if glaſſes are at hand, he may be better fitted by trial; taking notice to chuſe thoſe that are the leaſt convex or concave that will fit him. Theſe will prevent a perſon from growing more long-ſighted or ſhort-ſighted, for ſome time. Fig.

SCHOLIUM.

When glaſſes are put into frames for ſpectacles, theſe frames ought not to be ſtreight, ſo as both eyes may lie in the ſame plane; but they ought to be ſo bent in the middle, that the axes of both glaſſes may be directed to one point, at ſuch a diſtance as you generally look with ſpectacles. By this means the rays will fall perp. upon both glaſſes, and make the object appear diſtinct. But if they fall obliquely upon the glaſſes, it will cauſe a confuſed appearance in the objects. Therefore the ſhape of the frame ought to be as repreſented in fig. 105, where AB is the plane where one glaſs is fixt, and CD the other. 105.

Spectacles are the moſt uſeful optical machine in the world. Few people that grow into years, but have occaſion for them, to help their defective eyes, which without them would be uſeleſs, in a great many occaſions in life.

PROB. V.

To make a concave ſpeculum HD *of a given thickneſs* BD, *and whoſe inner radius* AB *is given; that ſhall reflect parallel rays* RH, *to the ſame focus* F, *from both ſurfaces* BH, DI. 106.

It is plain, that the ray which is reflected from the hindmoſt ſurface, will be twice refracted. For RH is firſt refracted at H to the focus *f*, and then

Fig. 106. reflected at I to the focus G, and laſtly refracted at L to the focus F.

Let AB $= a$, BD $= t$, DG $= x$, radius CD $= z$; m, n the ſines of incidence and refraction. Then to find f, the focus of the refracted ray HI; we ſhall have (by Cor. 1. Prop XIII. B. III.) $m - n : m$: : AB (a) : B$f = \frac{ma}{m-n}$; put $q = \frac{m}{m-n}$. Then to find the reflected ray IG, (by Prop. V. B. II.) Cf $(qa + t - z)$: CG $(z - x)$: : Df $(qa + t)$: DG (x). Whence $qax + tx - zx = qaz - qax + tz - tx$; and when reduced $x = \frac{qaz + tz}{2qa + 2t - z} =$ DG, and BG $= x - t = \frac{qaz + 2tz - 2qat - 2tt}{2qa + 2t - z}$. But (Prop. VI. B. II.) BF $= \frac{1}{2}a$. And ſuppoſing the refracted ray ILF to go back again, and be refracted along LI, whoſe focus is G; then (Cor. 1. Prop. XI. B. III.) $m \times$ BF $+ n \times$ AF : $m \times$ BF : : AB : BG. That is $\frac{1}{2}ma + \frac{1}{2}na : \frac{1}{2}ma : : a : x - t$, or $m + n : m : : a : \frac{qaz + 2tz - 2qat - 2tt}{2qa + 2t - z}$, and $\overline{ma} \times \overline{2qa + 2t - z} = \overline{m + n} \times \overline{qaz + 2tz - 2qat - 2tt}$. And putting $s = \frac{m}{m + n}$, we ſhall have $qaz + 2tz + saz = 2sqaa + 2sat + 2qat + 2tt$, and $z = \frac{2sqaa + 2sat + 2qat + 2tt}{qa + sa + 2t}$, and if m, and n be 3 and 2, then $z = \frac{9aa + 18at + 5tt}{9a + 5t}$.

Cor. 1. CD *or* $z = a + \frac{13}{9}t - \frac{tt}{4a}$ *very near; as will appear by diviſion; and therefore if* AD *or* $a + t$ *be ſubtracted, there will remain* AC $= \frac{4}{9}t$, *nearly.*

Cor. 2. *Hence alſo if* z *be given and* a *required, it may be found by reſolving the equation* $9az + 5tz = 9aa + 18at + 5tt$. *Or if* z *and* a *be given,* t *may be found from the ſame equation.* Fig. 106.

Cor. 3. *If ſuch a ſpeculum could be exactly ground, it would be better than any reflecting metal, not only becauſe it would reflect more light, but it would preſerve the poliſh longer. But the difficulty of executing them truly, has prevented the uſe of them.*

SCHOLIUM.

If inſtead of 3 and 2, 31 and 20 be put for m and n, as being more exact; we ſhall find z or CD $= \frac{2qs}{q+s}a + \frac{2qq+2ss}{\overline{q+s}^{2}}t$, nearly. And if $a+t$ be ſubtracted, then AC $= \frac{2sq-q+s}{q+s}a + \overline{\frac{q-s}{q+s}}^{2}t$, nearly. According to this computation, we ſhall have AC $= .416t$, which is not much different from the former, which was $\frac{4}{9}t$ or $.444t$.

The principal difficulty is to find the vertex of this reflector after it is made, or the point where it is the thinneſt. To do this, cover the ſpeculum with a paper, having a hole in it of the bigneſs the ſpeculum is to be when finiſhed, or when put into a teleſcope. Then it muſt be placed at a good diſtance from a candle, and the image received on a bit of paper juſt big enough to receive it. And moving the paper cover to all places of the ſurface, till you find ſuch a poſition for it, as makes the image the moſt diſtinct poſſible, there fix it; and the middle of the hole is the vertex. For tho' the two ſurfaces be ground very truly to their proper radii, yet if the centers are not in a right line, paſſing thro' the middle of the reflector, the two images

Fig. images cannot coincide, and they will confuſe o
106. another.

If a concave reflector be made after this ma ner, it will burn bodies in its focus more fierc than any other ſort; for both the reflected and r fracted rays meeting there, conſpire to make t effect greater.

PROB. VI.

107. *To make a teleſcope with a convex and a conca glaſs, to ſhew objects erect; called,* Galileo'*s* Tel cope.

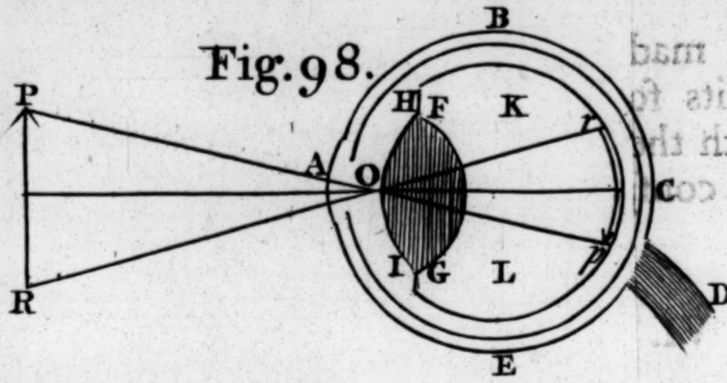
A Teleſcope is an optical inſtrument made by t combination of glaſſes, for ſeeing objects at a gre diſtance. The glaſs next the object, is called t *object glaſs*; and that next the eye, is the *eye gla* The *aperture* is the diameter of the hole throug which the rays paſs. And by reaſon of the gre diſtance of the object, all rays of the ſame penci incident on the object glaſs, muſt be eſteemed t be parallel; for the eye can diſcover no manner o difference between theſe and parallel rays.

To make this teleſcope, fix the convex lens A in the tube AEFB, and the concave lens CD i the tube CGHD, the concave lens muſt be of a leſs focal diſtance than AB, and the tube GD muſt be made to ſlide within the tube AF; and the glaſſes muſt be ſo placed, that the axes of the tubes and glaſſes may coincide; then ſliding the tube GD within the tube AF, till the diſtance of the glaſſes AC be equal to the difference of their principal focal diſtances, or that both their principal foci may coincide in the ſame point P; and then the teleſcope is fit for uſe.

The teleſcope being fitted thus, direct it with the axis RP ſtreight upon the object, and apply

the

Fig. 98.



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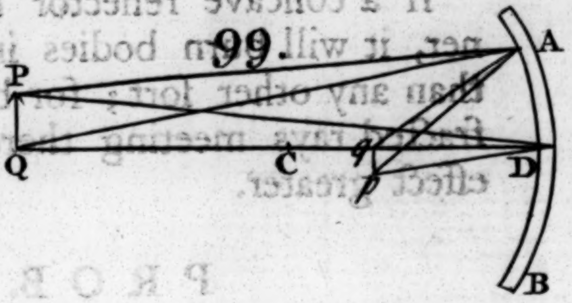

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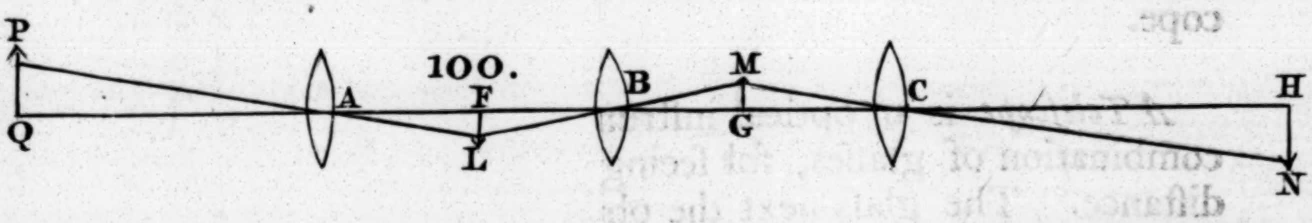
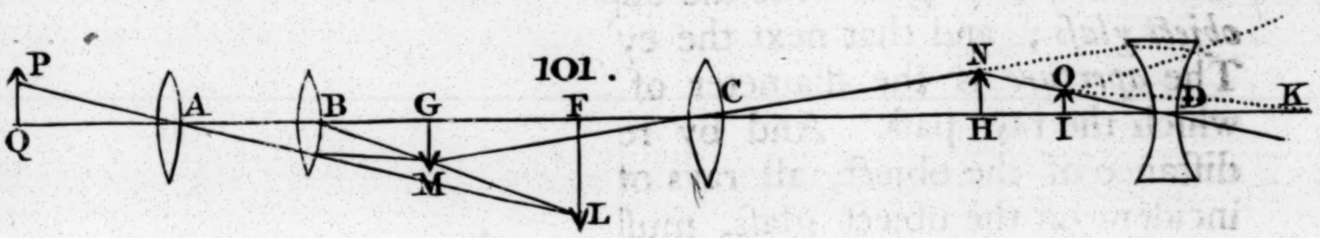
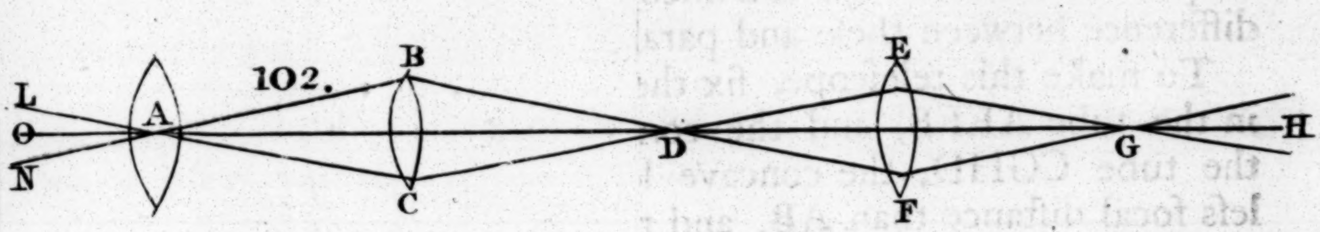


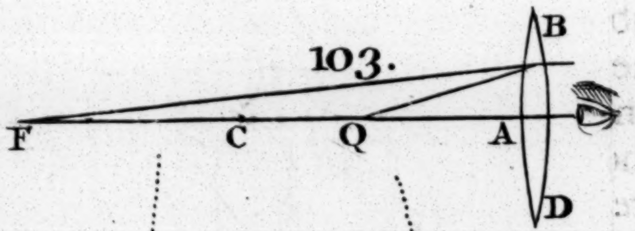
101.



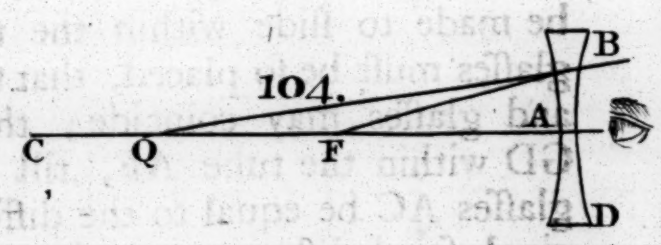
102.



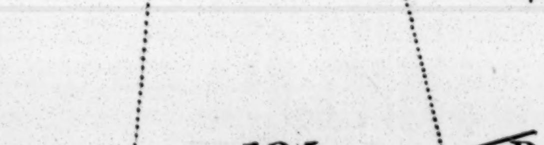
103.



104.



105.



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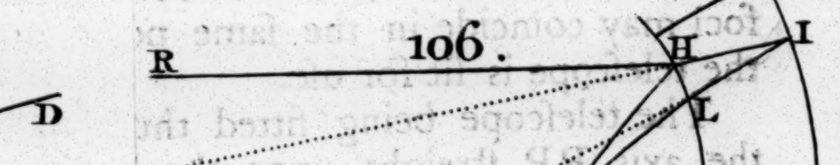

Opticks.

Pl. VIII. *pa. 170.*

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the eye cloſe to CD, and you'll ſee the object magnified, and erect. Fig. 107.

For ſince the rays proceeding from the object tend to the focus P; and before they arrive at it, are refracted by the concave CD; therefore they do not croſs one another before they come at the eye, and conſequently the image is erect. Or thus, ſince the conjugate focus to P (in the lens CD) is ſomewhere on the other ſide of CD, therefore if a line be drawn from Q, the lower part of the image at P (which is there inverted), thro' the middle of the lens CD, it will come to that conjugate focus above the axis, and therefore the image is erect.

The properties of this teleſcope.

1. The focal diſtance of AB muſt be greater than that of CD; or elſe it will not magnify an object. If AB be leſs, it will diminiſh it.

2. The magnifying power is equal to the quotient ariſing by dividing the focal diſtance of the object glaſs, by the focal diſtance of the eye glaſs $= \frac{AP}{CP}$.

3. The rays of any pencil coming out of the glaſs CD to the eye, are nearly parallel. If this does not ſuit all eyes, the tube GD muſt be put in, or drawn out a little, till the object appears diſtinct.

4. The viſible area of the object is the greater, the nearer the eye is to the glaſs; and depends on the breadth of the pupil of the eye, and of the object glaſs. Therefore the view will be ſmall.

5. In order to ſee more of the object, the eye muſt be moved laterally, or up and down, the teleſcope remaining fixt. By this means one may ſee ſucceſſively all the parts, that the teleſcope can take in at once, and that depends upon the breadth of the eye glaſs.

Fig. 107. 6. This ſort of teleſcopes muſt not be made long; for then the viſible area is ſo little, that they become uſeleſs.

For if you ſhut your eye for a while that the pupil may dilate, as it will in the dark; and then ſuddenly opening it, and looking ſteadily at an object thro' the teleſcope, you'll ſee it largeſt at firſt, and afterwards it gradually leſſens, as the pupil contracts by the light. Alſo if you cover the object glaſs with a paper with a hole in it; leſs of the object will appear than before; and ſo much leſs as the hole is leſs. And therefore the viſible area depends both upon the breadth of the pupil, and of the object glaſs.

Alſo if a paper be put over the eye glaſs, having a ſmall hole in it; leſs of the object will appear, by moving to and fro, ſideways; than did before, thro' the whole eye glaſs. And if the hole be leſs than the pupil, the direct view will be leſs.

In a long teleſcope of this ſort, the viſible area will be very ſmall, by the narrowneſs of the object glaſs. And if the object glaſs be made broader, the object will be coloured, and conſequenly confuſed.

This teleſcope is the ſame as is generally called a *common perſpective glaſs*.

PROB. VII.

108. *To make a teleſcope with two convex glaſſes, to ſhew objects inverted; called the* Aſtronomical Teleſcope.

Let A, B be two convex lenſes, HC, DE two tubes ſliding within one another. Fix the lens A with the longer focus, in the wider tube HC; and the lens B with the ſhorter focus, in the ſmaller tube DE; and let the axes of the glaſſes coincide with

ith the axis of both tubes. Then ſliding the ube DE, within the other, till the diſtance of the enſes be equal to the ſum of their principal focal iſtances; or that their principal foci may coincide n the ſame point P; and near P there muſt be a top, with a hole to circumſcribe the image, and o exclude all ſtraggling rays. Fig. 108.

And then the teleſcope is fit for uſing, which is one by directing it ſtreight to the diſtant object; nd then applying the eye beyond B, the diſtance f its focus; the object will appear magnified and nverted.

For the rays from the object R, croſſing at A, ill form the image at the focus P, inverted; hich viewed thro' the lens D, will therefore appear inverted. And P being the principal focus of B, the emerging rays will fall parallel upon the eye, nd cauſe diſtinct viſion.

The properties of this teleſcope.

1. The magnifying power is as the ratio of the focal diſtance of the object glaſs A, to that of the eye glaſs B.

2. The rays emerging out of the eye glaſs to the eye, muſt be nearly parallel, for a good eye. If this happens not to ſuit any other eye; then the tube DE muſt be thruſt in or pulled out, till the object appears diſtinct.

3. The apparent magnitude of the object is the ſame wherever the eye is placed, by Prop. 37. B. 3. But the viſible area is the greateſt, when the diſtance BO of the eye is nearly equal to BP, the focal diſtance of the eye glaſs. By Prob. III.

4. The object in this teleſcope is always inverted; and therefore it is only applied to aſtronomical uſes, to look at the celeſtial bodies. For it ſignifies little whether they appear erect or invert-

ed,

Fig. 108. ed, in theſe caſes. And they are commonly uſed in the night.

5. The viſual angle depends on the breadth of the eye glaſs. For it is equal to the angle which the eye glaſs ſubtends at the object glaſs.

PROB. VIII.

109. *To make a teleſcope with three convex glaſſes to ſhew objects erect.*

Combine the two glaſſes A, B, ſo that FA may be the principal focal diſtance of A; and FB twice the principal focal diſtance of B, and BP the ſame; let theſe be fixt in one tube. Then place the lens C, in another tube ſliding within the firſt, that PC may be its principal focal diſtance, which may be done by ſliding the inner tube back or forward; and then the teleſcope is finiſhed. The eye muſt be placed at the diſtance of 2PC behind C.

For parallel rays proceeding from the object R, and falling on the lens A, are refracted to the focus F, and there give the image inverted. And (Prop. XXI.) F, P are conjugate foci, and (Cor. Prop. XXIII.) they are at the leaſt diſtance poſſible. And ſince the image at F is inverted, that at P will be erect. And ſince PC is the focal diſtance of C, the rays falling on it, will emerge parallel, upon the eye; which therefore will ſee the object erect.

This teleſcope is of little uſe, by reaſon it takes in ſo little of the object; and makes the viſible area ſo ſmall.

SCHOLIUM.

A teleſcope may be made with two convex glaſſes to ſee objects erect. And that is by combining the two lenſes A, B, as before, which will give the image at P erect, which image may be ſeen

een by an eye placed ſix or eight inches behind it. Fig. This teleſcope is the ſame as the former, when the 109. glaſs C is taken away; but it takes not in ſo much of the object as the other, and therefore is of no uſe for teleſcopes.

A teleſcope may alſo be made with two convex lenſes, and a concave, which will ſhew objects inverted. Here the concave is the middlemoſt, being put inſtead of B. And they are placed thus, the foci of A and B muſt not coincide as in Prob. V; but they muſt be a little further aſunder, the focus of B being further from A than its focus is; and ſo far, that the conjugate focus (of B) may be at P; then if the eye glaſs C be placed, that its focus may be at P, then looking thro' the teleſcope the object will appear diſtinct, and inverted. This teleſcope, has full as great magnifying power as any refracting teleſcope; but becauſe the object is inverted, it is not uſed. The conſtruction of this teleſcope is the ſame as the ſolar teleſcope in Prob. XIII. following, fig. 114, having a concave eye glaſs L. For if *i* be its focus, and *b* the focus of AB; and *b*, K conjugate foci of L. Then K is the focus of a convex eye glaſs for this teleſcope.

PROB. IX.

To make a teleſcope with four convex glaſſes to ſhew objects erect.

Fix the lens A of a great focal diſtance in a 110. wide tube, and fix the three ſmall lenſes or eye glaſſes B, C, D in another tube, to ſlide within the former. The lenſes B, C, D, muſt be all of one focal diſtance, and muſt be placed equidiſtant in the tube, ſo that BC, and CD, may be twice the focal diſtance. Then if CP = PD, P will be the principal focus of C and D. A ſtop muſt be placed

Fig. placed at P or F, to ſtop the ſtraggling rays.
110. Then ſlide the tube with the three eye glaſſes, within the other, till the foci of A and B coincide in F. Then the teleſcope is made, then directing it to the object R, and placing the eye in the other principal focus of D; and the object will appear to it magnified and erect.

For parallel rays coming from the far diſtant object R, will be refracted by the object glaſs A to its focus F, and there make the image inverted. Then ſince F is the principal focus of B, the rays diverging from the image at F, and falling on the lens B will emerge parallel, and fall parallel on the glaſs C, and therefore will be refracted by it, to its principal focus P, and there form the image erect. Then the rays flowing from P the principal focus of D, will emerge a third time parallel, in falling on the eye; which (by Prob. III.) muſt be placed at the principal focus; and then the eye will diſtinctly ſee the image P of the object R, erect and magnified. Here then the firſt image is at F inverted, and the ſecond at P erect, from whence rays flow to the eye.

If BC be leſs than twice the focal diſtance of B or C, then ſome of the rays will miſs the eye glaſs D. And if BC be greater than twice the focal diſtance, ſome of the rays will miſs the glaſs C. And in either caſe the proſpect will be diminiſhed.

This teleſcope with four glaſſes is in effect two teleſcopes, of two glaſſes each, as in Prob. VII. The firſt teleſcope is AB, where the rays emerge parallel from B. The ſecond is CD, where the rays fall parallel on C, and emerge parallel from D. And ſince C and D are equal, they magnify nothing, and therefore the image at P is equal to the image at F. All that the glaſſes C, and D do, is to invert the image at F, or make the image of R erect. This is the common teleſcope uſed in the day time to look at diſtant objects. Fig. 110.

If a ſtop be placed at P, with a hole to contain the image; it will cut off the reſt of the rays which come irregularly, and would diſturb the diſtinctneſs of the image; and will take away the colours which ariſe by the different refrangibility of the rays, which would make the image confuſed. But the image will no more be coloured by four glaſſes than by two; for the glaſs C corrects and takes away the colours which the glaſs B produced.

The properties of this teleſcope.

1. The magnifying power, is in the proportion of the focal diſtance of the object glaſs, to the focal diſtance of the eye glaſs. This will be ſhewn afterwards.

2. It ſhews the object erect, but not ſo bright as in the teleſcope with two convex lenſes, becauſe the other two reflect a deal of the light which is loſt. In other reſpects it has the ſame properties as the other teleſcope.

3. The viſible area depends on the breadth of the eye glaſs, for as the eye glaſs is bigger or leſſer, the angle it ſubtends at A is greater or leſſer, or its oppoſite angle, which is the angle the part of the object ſubtends at A, which determines the viſible area. And this teleſcope takes in more than that in Prop. VI, with a concave eye glaſs.

4. The brightneſs of an object ſeen thro' a given teleſcope is ſo much the greater as the aperture of the object glaſs is greater. For ſuppoſe the object glaſs covered with paper, with a ſmall hole in the middle; the magnitude of the image in the focus will not be altered, nor conſequently that upon the retina of the eye. But the hole in the paper being ſmaller than the object glaſs, fewer rays of every pencil will come to the focus, and the image will be more faint.

Fig. 110. 5. If the aperture and focal diſtance of the object glaſs remains the ſame; an object appears brighter or fainter, according to the greater or leſſer focal diſtance of the eye glaſs; that is according to the leſſer or greater magnifying power. For the ſame quantity of light ſpread over a ſmaller or greater image, makes it brighter or duller.

6. It is found by experience, that an object glaſs of 55 feet focal diſtance will bear an aperture of 4 inches, and an eye glaſs, whoſe focal diſtance is 4.4 inches. And for other object glaſſes, the apertures thereof, and the focal diſtances of the eye glaſſes, muſt be as the ſquare roots of the focal diſtances of the object glaſſes. But to view objects by day light, requires eye glaſſes whoſe focal diſtances are longer; and by night ſhorter.

SCHOLIUM.

It may be thought, that in a teleſcope of any length, an object may be magnified in any proportion, by making the focal diſtances of the object glaſs and eye glaſs in that ratio. But there are two cauſes which prevent this. 1. Whilſt the object glaſs remains the ſame, the more the object is to be magnified, the more convex the eye glaſs muſt be, and then the more obſcure and dark the object will appear. 2. By this means the object will be repreſented the more confuſedly; and if one attempts to remedy this by increaſing the aperture, the confuſion will be more increaſed, and the objects will at laſt appear coloured. So that increaſing the aperture of the object glaſs, makes the object appear brighter, but at the ſame time it is confuſed and indiſtinct.

Since the pupil dilates itſelf very much in the dark, therefore there is not occaſion for ſo much light in looking at the moon or ſtars.

There are teleſcopes made with five or ſix or more glaſſes, which by refracting the rays gradually to a focus, will bear a larger aperture; but then the object is more faint, by the loſs of ſo many rays, by the reflection from ſo many glaſſes. Fig. 110.

Sometimes the eye glaſs D is of a different focus from B and C. But the diſtance BC muſt ſtill be the ſum of the focal diſtances of C and D. Thus ſhorter eye glaſſes are uſed in the night, to increaſe the brightneſs of the object.

PROB. X.

To find the magnifying power of a teleſcope.

Let the far diſtant object appear from A under the angle RAS, or its oppoſite GAF, drawn thro' the center of the object glaſs A. And let B be eye glaſs, FG the image in the focus of the glaſſes A and B. Then thro' the extremes F, G, of the image, draw the lines FBL, GBN, thro' the center of the lens B, and BL, BN will be the refracted rays (by Prop. XVII. B. III.) Then the object is ſeen thro' the teleſcope under the angle LBN or FBG, and it is ſeen from A or from B under the angle RAS or GAF. Therefore the apparent angle thro' the teleſcope, is to the apparent angle to the naked eye; as angle GBF, to the angle GAF; that is as AF to BF; that is, as the focal diſtance of the object glaſs, is to the focal diſtance of the eye glaſs. And if there be four convex glaſſes; if the glaſſes B, C, D, are all equal (fig. 110), the image at P is equal to the image at F, and the magnifying power continues the ſame as before. But if they be different glaſſes, the image at F will be to the image at P, as the focal diſtance of B, to the focal diſtance of D. So that at laſt, the apparent angle thro' the teleſcope, will be to the 111. 112.

Fig. apparent angle to the naked eye; as the focal diſ-
111. tance of the object glaſs, to the focal diſtance of
112. the laſt eye glaſs. See alſo the Corollaries of
Prob. II.

Cor. 1. *The magnifying power by a refracting teleſcope, is equal to the focal diſtance of the object glaſs divided by the focal diſtance of the eye glaſs.*

111. Cor. 2. *In a teleſcope with convex glaſſes; the viſual angle or linear view, is as the diameter of the eye glaſs, divided by the focal diſtance of the object glaſs. And the viſible area as the ſquare of that quantity.*

This depends upon the breadth of the eye glaſs, therefore drawing AD to the edge of it, BAD will be half RAS or half the viſual angle; and that angle BAD or its double is as $\frac{BD}{BA}$; and the viſible area as $\frac{BD^2}{BA^2}$. In the teleſcope with a concave glaſs (fig. 112.) it depends on the breadth of the pupil.

Cor. 3. *As the diſtance between the object glaſs and eye glaſs, to the breadth of the eye glaſs; ſo is radius, to the tangent of the angle which the teleſcope takes in.*

For all the teleſcope can take in, is contained between two lines drawn from the edges of the eye glaſs, through the center of the object glaſs. The contained angle is what the teleſcope can take in.

PROB.

PROP. XI.

To find the degree of indiſtinctneſs, or confuſed appearance of an object, ſeen thro' a teleſcope.

Since no lens can be made ſo perfect, as to refract all the rays iſſuing from one point of an object, to a ſingle point in the picture, but there will always be a deviation or aberration more or leſs, according as the lens is broader or narrower. And by reaſon of this aberration, the rays proceeding from one point will be ſpread over a certain ſpace of the image, and cauſe that confuſion or indiſtinctneſs obſerved in teleſcopes; which ſpace is the circle of aberration in the focus of the object glaſs. And this confuſion will be ſo much the greater, as that circle is larger, becauſe there are ſo many more different points of that ſpace, which repreſent but one point of the object; and therefore that confuſion or indiſtinctneſs is, *ceteris paribus*, as the circle of aberration in the focus of the object glaſs; that is, by (Cor. 5. Prop. XLI. B. III.) as the area of the object glaſs.

Now the confuſion of the image in the bottom of the eye (which is that we ſeek) is as the apparent magnitude of this circle of aberrations. And the apparent diameter of aberration ſeen thro' the eye glaſs, is as the angle made by the extreme rays in falling upon the eye, or (which is equal to it, by Cor. 2. Prop. XVII. B. III.) as the angle ſubtended by that diameter at the center of the eye glaſs, that is, as the diameter itſelf directly, and the focal diſtance of the eye glaſs reciprocally. Therefore the apparent magnitude of the circle of aberrations, is as the ſquare of that diameter directly, and the ſquare of the focal diſtance of the eye glaſs reciprocally; or as the ſquare of the aper-

Fig. ture of the object glaſs directly, and the ſquare of the focal diſtance of the eye glaſs reciprocally. And therefore the confuſion or indiſtinctneſs is in that ratio.

Cor. 1. *The indiſtinctneſs of an object ſeen thro' a refracting teleſcope, is as the ſquare of the aperture of the object glaſs, divided by the ſquare of the focal diſtance of the eye glaſs.*

Cor. 2. *An object appears equally diſtinct in ſeveral refracting teleſcopes, when the diameters of the apertures of the object glaſſes, are in a given ratio to the focal diſtances of the eye glaſſes.*

PROB. XII.

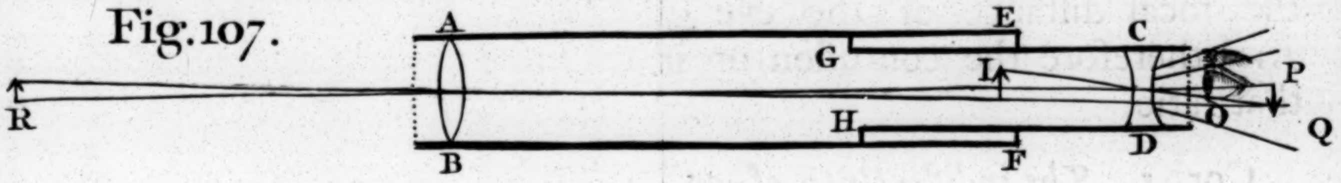
To find the apparent brightneſs of an object, ſeen thro' a refracting teleſcope.

If the area of the image upon the retina be the ſame, the brightneſs will be as the quantity of light upon it, that is, as the area of the aperture of the object glaſs. And if the aperture or quantity of light be the ſame, the brightneſs will be reciprocally as the area of the image, that is, reciprocally as the ſquare of the magnifying power. And therefore in general the brightneſs is as the area of the object glaſs directly, and the ſquare of the magnifying power reciprocally.

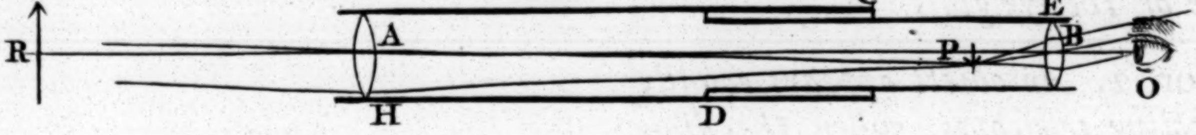
Otherwiſe thus.

113. Let two pencils of rays falling on the object glaſs CD, be refracted to the foci F, *f*; and paſs thro' the eye glaſs B*b*; and be refracted in the directions BL, *b*l. Let BL = FB, then L is the place of the eye, being the place where the ſeveral

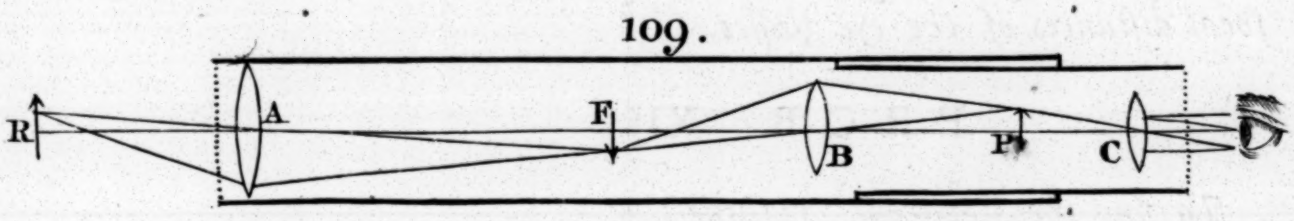
Fig. 107.



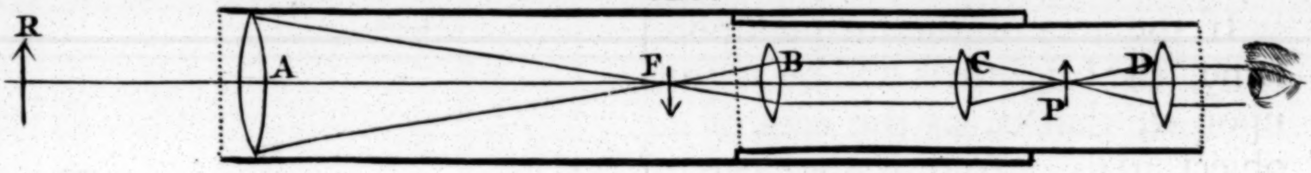
108.

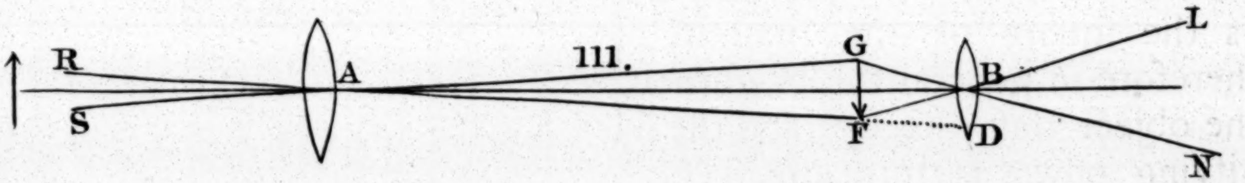


109.

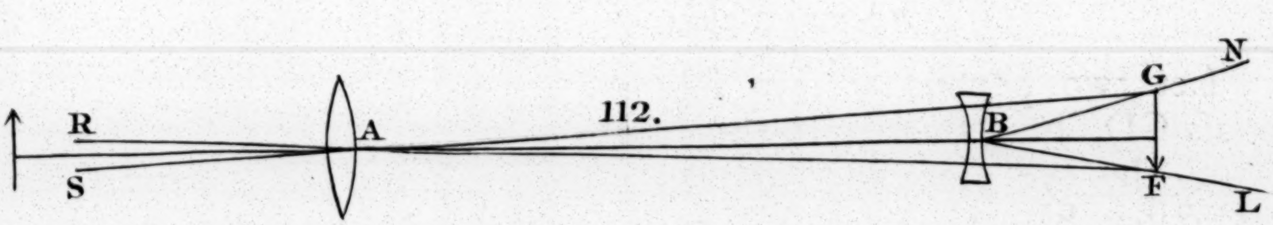


110.





111.



112.

Opticks. Pl. IX. *pa* 182.

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ral pencils (each conſiſting of parallel rays) converge to, and interſect, after refraction at B*b*. Fig. 113.

Let m = magnifying power, p = diameter of the pupil. Then by ſimilar triangles, AF : FB : : CD : B*b* or LI the breadth of any pencil of rays at B, or at L where the eye is; becauſe beyond B, the rays of each pencil are parallel. Therefore the breadth $LI = \frac{FB \times CD}{AF} = \frac{CD}{m}$.

Let the degree of common light be = 1, the area of the picture in the naked eye = a, then thro' the teleſcope it will be = mma. Then ſince the brightneſs is as the area of the aperture divided by the area of the picture, the brightneſs of the object by the naked eye will be $\frac{1 \times pp}{a}$; and by the teleſcope, $\frac{CD^2}{mma}$. Therefore common light : to teleſcopic light : : $\frac{pp}{a} : \frac{CD^2}{mma} :: 1 : \frac{CD^2}{mmpp}$. And ſince the common light is 1, the teleſcopic light will be $\frac{CD^2}{mmpp}$.

Now ſuppoſe LI to be juſt equal to p. Then $pp = \frac{CD^2}{mm}$, and in this caſe the teleſcopic light becomes 1, and is therefore equal to common light.

But diminiſhing CD, LI is alſo diminiſhed, and therefore in this caſe, leſs light enters the pupil; and therefore in all caſes where LI is leſs than p, the brightneſs = $\frac{CD^2}{mmpp}$.

But if CD be increaſed never ſo much, the brightneſs is not increaſed at all by the ſide rays, becauſe they will all miſs the pupil. For as CD increaſes in breadth, B*b* and LI alſo increaſe in breadth, and will therefore, in this caſe, be bigger

than

Fig. 113. than the pupil. And hence the denſity of the light is the ſame as before. For there is a greater quantity of light on a greater ſpace CD, or on a greater ſpace B*b* or LI; and the quantity of light being as the ſpace, the denſity is not increaſed.

By ſome experiments, I have found $\frac{1}{6}$ part of the light loſt in paſſing thro' two glaſſes; and near $\frac{1}{3}$, in paſſing thro' four, and more ſtill in paſſing thro' five or ſix. This happens by the reflexion of the two ſurfaces of every lens, which reflect a part of the light, that is loſt; and ſo much is the brightneſs decreaſed.

Therefore in general the brightneſs is increaſed, by increaſing the object glaſs, till LI becomes equal to the breadth of the pupil; but no longer.

Cor. 1. *If an object is ſeen thro' a refracting teleſcope, and the rays do not fill the pupil; its apparent brightneſs is as the ſquare of the aperture of the object glaſs, multiplied by the ſquare of the focal diſtance of the eye glaſs, and divided by the ſquare of the focal diſtance of the object glaſs.*

This follows from the firſt method. For the magnifying power is as $\frac{AF}{BF}$; and therefore the brightneſs is as CD^2, divided by $\frac{AF^2}{BF^2}$, or as $\frac{CD^2 \times BF^2}{AF^2}$. Or by the ſecond method, the brightneſs is as $\frac{CD^2}{mmpp}$, p being given.

Cor. 2. *According to this; an object appears equally bright, thro' ſeveral teleſcopes, when the apertures are as the magnifying powers.*

Cor. 3. *The ſame thing ſuppoſed; in different teleſcopes, an object will appear equally bright and diſtinct,*

tinct, when the apertures (of the object glaſſes), and focal diſtances of the eye glaſſes, are each of them as the ſquare roots of the focal diſtances of the object glaſſes. And then the magnifying powers will alſo be as the ſquare roots of the focal diſtances of the object glaſſes. Fig. 113.

For put F, f for the focal diſtances of the object glaſs and eye glaſs; and that the brightneſs and diſtinctneſs may be in the ſame ratio, (by Cor. 2. of the laſt Prop. and Cor. 2. of this), we ſhall have the aperture $\propto f \propto \frac{F}{f}$, whence $F \propto ff$, and $f \propto \sqrt{F}$, conſequently the aperture $\propto \sqrt{F}$. And laſtly, the magnifying power $\propto \left(\frac{F}{f} \propto\right) \frac{ff}{f}$ or as f, that is as $\sqrt{F}$.

Cor. 4. *In ſimilar teleſcopes; the diſtinctneſs, viſual angle and magnifying power are the ſame; and the brightneſs as the ſquare of the length.*

For let D be the diameter of the aperture; F, f the focal diſtances of the object glaſs and eye glaſs; then the diſtinctneſs is $\frac{D}{f}$, which is a given ratio. And the magnifying power is $\frac{F}{f}$, which is alſo a given ratio. And the viſual angle is as the diameter of the eye glaſs divided by F, which is a given ratio. And laſtly, the brightneſs is as $\frac{CD^2}{mm}$ or $\frac{DD}{mm}$, and m or the magnifying power is given; and the aperture CD is as the length of the teleſcope.

Cor. 5. *Hence, no teleſcope can ſhew an object ſo bright as it is to the naked eye. The neareſt it can approach to it is $\frac{5}{6}$ the light for two glaſſes, and $\frac{2}{3}$ for four glaſſes.*

For

Fig. 113. For when all the rays enter the eye, the teleſcopic light, or what paſſes thro' the teleſcope into the eye, is $\left(\frac{CD^2}{mmpp} = \right)$ 1, or the ſame as common light. Of which $\frac{1}{6}$ or $\frac{1}{3}$ is to be deducted for reflected rays, which are loſt.

Cor. 6. *According to what has been delivered before (after Prob.* IX. *and in Cor.* 3 *of this), the focal diſtances and apertures muſt be made according to the following table.*

foc. obj. glaſs	aper. obj. glaſs	foc. eye glaſs	mag. power	foc. obj. glaſs	aper. obj. glaſs	foc. eye glaſs	mag. power
feet	inch	inch		feet	inch	inch	
1	.54	.60	20	35	3.20	3.52	119
2	.76	.83	28	40	3.41	3.75	128
3	.94	1.03	34	45	3.62	3.98	135
4	1.08	1.19	40	50	3.82	4.20	143
5	1.22	1.34	44	55	4.00	4.40	150
6	1.32	1.45	49	60	4.18	4.60	156
7	1.43	1.57	52	70	4.52	4.97	169
8	1.53	1.68	56	80	4.83	5.31	181
9	1.62	1.78	60	90	5.13	5.64	192
10	1.71	1.88	64	100	5.40	5.94	202
13	1.95	2.14	73	120	5.91	6.50	222
15	2.09	2.30	78	140	6.39	7.03	239
20	2.42	2.66	90	160	6.83	7.51	255
25	2.70	2.97	101	180	7.25	7.97	271
30	2.96	3.25	110	200	7.64	8.40	216

SCHOLIUM.

It follows from Cor. 3, that if a parabola be deſcribed, and in the axis thereof, taking the abſciſſa's equal to the focal diſtances of the object glaſſes; then the correſpondent ordinates will be proportional to the apertures, or the focal diſtances of the eye glaſſes, or the magnifying powers. Fig. 113. For theſe are as the ſquare roots of the abſciſſæ, that is, as the ſquare roots of the focal diſtances of the object glaſſes. And inſtead of the focal diſtances of the object glaſſes, one may ſubſtitute the length of the teleſcope, without ſenſible error.

In the inveſtigation of the foregoing Problems, the indiſtinctneſs ariſing from the aberrations cauſed by the eye glaſſes are ſet aſide as being inconſiderable. For in ſo ſhort a diſtance as from the eye glaſs to the eye, the heterogeneous rays cannot ſeparate ſo as to become ſenſible, and therefore the errors may be lookt upon as nothing. And the refractions at ſo ſmall a diſtance will be ſufficiently true and regular.

To try the goodneſs of an object glaſs; proceed according to the ſecond method laid down in Prop. XVI. B. III. And if all the holes unite in one, the glaſs is good, otherwiſe not. But there is no better way for trying the goodneſs of an object glaſs than putting it in a tube, and trying it with ſeveral ſmall eye glaſſes, by looking at ſeveral diſtant objects, and particularly at the title page of a book. For that glaſs which repreſents objects the moſt bright and diſtinct, and bears the greateſt aperture, and the ſhorteſt eye glaſs, without colouring or dimneſs, is the beſt glaſs.

If ſeveral teleſcopes of the ſame length be compared together, theſe are the beſt, with which you can read the ſame print, at the greateſt diſtance. And this may be a rule for thoſe that buy teleſcopes, by which they may know how to chuſe the beſt.

There are four principal things in a teleſcope to be conſidered. 1. Magnifying power. 2. Diſtinctneſs. 3. Brightneſs. 4. The viſible angle, or linear view it takes in. And all theſe have

Fig. been determined in the foregoing Problems, for refracting teleſcopes.

PROB. XIII.

To make a ſolar teleſcope; or to fit up a teleſcope, for obſerving eclipſes, ſpots of the ſun, &c.

114. This is no more than a common refracting teleſcope with two glaſſes, where the eye glaſs may be either convex or concave. Thro' this teleſcope the ſun's image is projected on a plane ſurface, or on the oppoſite wall of a dark room, and magnified. If a teleſcope be uſed for this, as it is fitted up for looking at diſtant objects, the ſun's image will be projected at an infinite diſtance, becauſe the rays emerge parallel out of the eye glaſs. Therefore to have his image at any given diſtance, the glaſſes muſt he placed after this manner.

Let ABCD be a tube in which is the object glaſs G, and its focus H. ECDF another tube ſliding within the former, in which is the eye glaſs L, whoſe focus is I. Let K be the place where the ſun's image is to fall, upon the plane MN. Now whether the glaſs L be convex or concave, it muſt be placed further from G, than in the common teleſcope, by the diſtance HI. And H, K muſt be the conjugate foci of the glaſs L, which is done by making HI, HL, HK in continual proportion. Or if L be a concave, h the focus of G, i the focus of L, then hi, hL, hK muſt be in continual proportion. Then if the teleſcope AD be put thro' a hole in the window ſhut of a dark room, and directed to the ſun, you'll have his image at K, upon the plane MN. Or if the plane MN be fixt, then ſlide the tube CF, back and forward, within the tube FA, till you find the image diſtinct at K, upon that plane.

For

Fig. 114. For if H be the focus of the lens AB, the ſun's image will be at H inverted. And as H, K, are conjugate foci; the image of K or the ſecond image of the ſun will be at K erect. But in the concave lens CD, the firſt image would be at *h*, and the ſecond at K inverted.

Cor. 1. *To find the diameter of the image* MN; *it will be as* HL : LK : : *as the diameter of the image at* H : *to* MN *the diameter of the image at* K.

For the ſun and the image at H, ſubtend equal angles at the center of the lens G (by Prop. XXXI. of B. III.); and likewiſe the image at H and the image at K, ſubtend equal angles at the lens L; whence the foregoing proportion.

Cor. 2. *The larger the image* MN *is made, the fainter it will be.*

For if it be as big as the object glaſs G, it will be as bright as if the ſun ſhined upon it. And therefore if it be bigger, its brightneſs will be reciprocally as the area.

Cor. 3. *After this manner may be viewed the ſun's face, his ſpots, his eclipſes, the tranſits of the planets, clouds; likewiſe the moon's face, &c. And this teleſcope is uſed by aſtronomers for this purpoſe.*

PROB. XIV.

115. *To fit up a teleſcope for taking ſmall angles, by placing croſs hairs in the focus of the eye glaſs.*

Take a braſs plate AB, in which make a round hole, whoſe diameter is ſomething leſs than that of the eye glaſs; this being placed in the focus P of the eye glaſs (fig. 108, 110.), ſerves for a ſtop to cut off the ſuperfluous rays; and in this the hairs are to be fixt, or rather ſeveral pieces of ſmall ſilver wire. In this plate is fixt the ſmall wire *pq*, going

thro'

Fig. 115. thro' the center C of the circular hole; and perpendicular to it is fixt the hair *rs*, near the ſide of the hole. There is a ſlider BDE, in which is fixt the hair or wire *tv*, parallel to *rs*, ſo that when the ſlider is thruſt up cloſe to the ſide EB, the hair *tv* may coincide with *rs*; and when it is drawn out to the extent, the hair may be near the other ſide of the hole, at EB. Then the ſpace BF is divided into 10 or 100 parts, which is the utmoſt extent between the hairs *rs* and *tv*. Then the plate ADE muſt be fixt exactly in the plane of the image made at P (fig. 108, 110,) which may eaſily be done after a few trials; for when it is right, the hairs will appear very diſtinctly thro' the eye glaſs, then they muſt be covered with a thin lantern horn. When the place of the plate is had, one may find the focus of the object glaſs by meaſuring from the hairs to the glaſs: and this is the exacteſt way. The plate muſt alſo be placed ſo, that the center C of the hole may be in the axis of the teleſcope. Then we muſt find what arch is repreſented by BF or *no*, the extreme diſtance of the hairs *rs* and *tv*, when the ſlider is drawn out to its uttermoſt.

To find the arch or angle repreſented by no.

Here the croſs hairs muſt be illuminated by a candle, made to ſhine on them. Direct the teleſcope to ſome ſtar, and turn it round, till you find the ſtar moves along the hair *pq*; then the teleſcope is in a right poſition; therefore keeping the ſame ſide uppermoſt; draw out the ſlider to the end of the diviſions, and direct the teleſcope ſo to the ſtar, that it may fall upon the interſection of the croſs hairs at *n*. Then the teleſcope being fixt there, obſerve by a clock or a watch, how long the ſtar is moving from *n* to *o*. Then ſince one degree paſſes thro' any point in four minutes time, or fifteen minutes of the equinoctial in one minute of time or ſixty ſeconds. Whence one ſecond of time correſponds to fifteen ſeconds of meaſure. Therefore dividing the number of ſeconds obſerved by the clock by four, gives the minutes of a degree anſwering to *no*. But if the ſtar is out of the equinoctial, this number muſt be leſſened in the ratio of radius to the coſine of the ſtar's declination. Fig. 115.

Otherwiſe.

Set up two marks againſt a wall at a good diſtance from you, ſo that looking thro' the teleſcope, the two hairs *rs*, *tv*, ſet to their utmoſt extent, may juſt cover the two marks. If they do not, ſhift the teleſcope further back or nearer hand; or elſe ſhift the marks, till they do. Then meaſuring the diſtance of the teleſcope, as likewiſe the diſtance of the two marks from each other, you will find by plane trigonometry the angle included by the viſual rays; and that is the angle repreſented by *no*.

To take an angle with the teleſcope.

Direct the teleſcope, ſo that the fixt hair *rs* may juſt touch one ſide or term of the angle to be meaſured; and draw out the ſlider EF, till the moveable hair *tv* touches the other ſide; then mark the diviſion of the ſcale at B; and knowing before what arch *no* repreſents: find by the rule of three what angle the number of the ſcale at B amounts to.

If there be no moveable ſlider, then the two hairs *rs*, *tv*, muſt be both fixt. But then in taking an angle, when one ſide is ſet to *rs*, the other ſide will interſect *no* in ſome point, whoſe diſtance from *n*, in reſpect of *no*, can only be taken by gueſs, as there will be no ſcale to meaſure it by.

But the moſt exact way of all for taking ſmall angles, is by the *micrometer*, which is an inſtrument, Fig. 115. ment, with the croſs hairs like theſe deſcribed; and beſides there is a machine made of clock-work, that by turning the handle, moves the hair *tv* always parallel to itſelf; and by one or two indexes, ſhews the diſtance of the hairs very exactly.

There is another way of fixing the hairs, by placing them diagonally, making half a right angle with one another. And theſe are of uſe for finding the difference of right aſcenſion, or of declination of any two ſtars, which the teleſcope can take in.

Theſe croſs hairs, and micrometers, are only applied to refracting teleſcopes, but have not yet been uſed with reflecting ones, ſince they do not magnify the angle ſo much.

Cor. 1. *Hence the diameters of the planets, may be truly meaſured; and all the particulars of an eclipſe determined; or any ſmall angles accurately taken; and applied to other aſtronomical uſes.*

Cor. 2. *The line* no *ſubtends an angle at the eye, which is greater than the angle it repreſents; in proportion to the magnifying power of the teleſcope.*

For the line *no* being placed with the image, they are both magnified alike.

Cor. 3. *Hence,* no *repreſents the angle which it ſubtends at the center of the object glaſs.*

For it is magnified in the ratio of the focal diſtance of the object glaſs to the focal diſtance of the eye glaſs; that is, as the angle it ſubtends at the eye glaſs, to that it ſubtends at the object glaſs.

Fig. 115.

PROB. XV.

To manage teleſcopes without tubes.

To do this, a long pole muſt be ſet upright in the earth, in a wide place, where there is nothing to obſtruct the ſight. In this pole is made a grove, in which a piece of wood is to be fitted to ſlide freely up and down, this is to be drawn up by a rope going over a pulley at the top, when it is uſed. The object glaſs is put into a ſhort tube AB, and this tube is fixt to a piece of machinery, made to move about two axes CD, lying acroſs one another, and this machinery is fixed to the ſliding piece of wood. On the under ſide of the tube a ſmall rod AF is fixt parallel to it, reaching ſix or eight inches beyond the tube; to the end of this a ſtring G is tyed, reaching to the ground. Then any perſon on the ground pulling at the ſtring, can draw the tube into a poſition parallel to the ſtring, by means of the croſs joints; ſo that the tube will point directly to the perſon that draws the ſtring.

The eye glaſs is alſo put in a ſhort tube H, on the under ſide of which is alſo fixt a ſmall rod, at the far end of which I, the low end of the ſtring is to be fixt; and at the nearer end a handle K is fixt to take hold of.

When an obſervation is to be taken, the ſliding piece of wood, with the object glaſs, is to be drawn up the pole to a proper height, and there fixt. Then drawing the ſtring IG, which is fixt to the tail of the object tube, to ſet it ſtreight on you; tie the low end of it to the end of the ſmall rod at the eye tube, ſo as the length of it between the glaſſes may be of a due length for diſtinct viſion, which in the aſtronomical teleſcope, is equal to the ſum of the focal diſtances. Then the ſtring

Fig. 115. drawing the two tubes, ſets them in a right line, and if they alſo be in a right line with the ſtar you want to obſerve, then applying your eye to the tube, you will ſee the ſtar thro' the glaſſes. But if they are not in a line, you muſt alter your poſition, till they are ſo.

It is neceſſary that the obſerver have a reſt to lean on, or to lay the eye tube upon, like a chair back. For without it, the tube cannot be held ſteady. And the obſervations muſt be made in calm fair weather, for the wind will ſhake the tubes, and prevent any thing from being ſeen diſtinctly. And beſides in windy weather, the air is always foul, which makes it unfit to make any obſervations in. And the ſame thing often happens in calm weather, when the air is full of groſs aqueous vapours.

And after this manner perſpectives may be uſed without cloſe tubes; which may be done by holding the eye tube, applied to one end of a long ſtick, with one hand; and the object tube in the other hand, applied to the other end of the ſtick; and ſliding it back and forward on the ſtick to a proper diſtance; which is ſoon found by a few trials. Obſerving to keep the tubes ſtreight on one another, and parallel to the ſtick. After this manner one may uſe the glaſſes of a perſpective as long as ones arm, or longer, by tying the object tube to the ſtick, at a proper diſtance from the nearer end of it; and after this manner, the goodneſs of glaſſes may be tried, without the trouble of putting them into long tubes.

And the ſame way one may make uſe of a long moveable pole, inſtead of one fixt in the ground; which may be uſed for glaſſes not exceeding ſix or eight yards focal diſtance, by tying the eye tube to one end, and the object tube towards the other, at a proper diſtance. And it may be raiſed upon a wall, or the ſide of a houſe, by the help of a Fig. hay fork, or ſome ſuch thing. 115.

Sometimes the ſight is diſturbed, by the ſide light coming into the eye tube, which a cloſe tube would have kept out. To prevent which, two ways may be uſed; either to lengthen the tube of the eye glaſs; or to put an umbrella or border round the tube with the object glaſs; and then to take ſight truly at the object, you muſt raiſe your eye above the eye tube, as high as the breadth of the umbrella.

To enlighten the object tube, to take ſight; a lantern may be uſed, tied to the end of a long pole, and ſo raiſed up on high.

PROB. XVI.

To make a reflecting teleſcope.

A reflecting or catadioptric teleſcope may be made ſeveral ways. I ſhall firſt deſcribe the *Gregorian* one, becauſe they are moſt in uſe.

The Gregorian Teleſcope.

ABCD is a tube open at the end AB, FG a 117. concave reflector, having a hole in the middle, and whoſe focus is *n*. *arb* a ſmall concave ſpeculum, whoſe focus is *t*. This is ſo placed, that the focus *n* of the great reflector, and the place of the image *o*, may be conjugate foci, or that *tn*, *tr*, *to* may be continual proportionals. HI is a ſhort braſs tube, ſcrewing into the end of the large one. In this tube is the convex lens E, ſo placed, that EO may be its focal diſtance, and EI the ſame; and at the end I is a ſmall hole for the rays to come thro' to the eye. But I or the place of the eye is found more exactly, by making *ro*, *r*E, *r*I continual proportionals.

Fig. 115. drawing the two tubes, ſets them in a right line, and if they alſo be in a right line with the ſtar you want to obſerve, then applying your eye to the tube, you will ſee the ſtar thro' the glaſſes. But if they are not in a line, you muſt alter your poſition, till they are ſo.

It is neceſſary that the obſerver have a reſt to lean on, or to lay the eye tube upon, like a chair back. For without it, the tube cannot be held ſteady. And the obſervations muſt be made in calm fair weather, for the wind will ſhake the tubes, and prevent any thing from being ſeen diſtinctly. And beſides in windy weather, the air is always foul, which makes it unfit to make any obſervations in. And the ſame thing often happens in calm weather, when the air is full of groſs aqueous vapours.

And after this manner perſpectives may be uſed without cloſe tubes; which may be done by holding the eye tube, applied to one end of a long ſtick, with one hand; and the object tube in the other hand, applied to the other end of the ſtick; and ſliding it back and forward on the ſtick to a proper diſtance; which is ſoon found by a few trials. Obſerving to keep the tubes ſtreight on one another, and parallel to the ſtick. After this manner one may uſe the glaſſes of a perſpective as long as ones arm, or longer, by tying the object tube to the ſtick, at a proper diſtance from the nearer end of it; and after this manner, the goodneſs of glaſſes may be tried, without the trouble of putting them into long tubes.

And the ſame way one may make uſe of a long moveable pole, inſtead of one fixt in the ground; which may be uſed for glaſſes not exceeding ſix or eight yards focal diſtance, by tying the eye tube to one end, and the object tube towards the other, at a proper diſtance. And it may be raiſed upon a wall, or the ſide of a houſe, by the help of a hay fork, or ſome ſuch thing. Fig. 115.

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Fig. 117. Now the open end AB being ſet directly towards the object, the rays coming from all the points of the object, are parellel; and being reflected by the concave ſpeculum FG, will form the image of that object at *n*, which will be inverted (by Prop. XVIII. B. II.) And for the ſame reaſon, rays flowing from *n*, and reflected by the ſmall concave *ab*, will make the ſecond image at *o*, which is inverted in reſpect of the firſt image, and erect in regard to the object. Laſtly, the rays flowing from the image at *o*, which is the focus of the glaſs E, and being refracted by it, will emerge parallel to the eye; and therefore the object will appear erect thro' the teleſcope. The path of an oblique ray coming from the upper part of the object, is KF*xbsw*I.

To ſuit all ſorts of eyes, there is a ſmall rod ML, with a ſcrew at the end M; this ſcrew goes thro' the arm which is fixt to the ſmall reflector *ab*, ſo that turning the end L it brings it nearer, or ſets it further, from the other reflector FG. It has a ſhoulder N to keep it ſteady.

The two ſpeculums FG, *ab*, are made of a mixture of copper and tin, but any white hard metal will do that will take a good poliſh. But no mixture has been found out yet that will poliſh truly without pores, ſcars, and ſcratches, and dark ſpots in them. And the ſpeculum *ab* muſt be a little bigger than the hole in the middle of the ſpeculum FG.

118. Inſtead of a ſingle eye glaſs, they are now generally made with two; which increaſes the viſible area a little, and prevents the object being coloured at the edges of the aperture. In order to this, the conjugate focus *o*, muſt be taken in the leſſer tube HI, a good diſtance beyond the hole; and then place a lens P near the hole, to make the rays converging to *o*, to converge ſooner, as to *q*; and

Fig. 118. and there to form the image. And then place another lens (a meniſcus) at E, ſo that Eq and Ev may equal the focal diſtance of it. Then the image which before was at o, by the refraction of the lens P, will now be at q. Therefore rays flowing from the image at q, and refracted by the lens E, will emerge parallel to the eye at v. But there is one inconvenience attending two eye glaſſes, that the object will be fainter by ſo many reflexions from their ſurfaces.

An eye ſtop is to be placed at the laſt image q, to circumſcribe the image, and to cut off the ſuperfluous rays, which is a thin baſs plate, with a hole in it, leſs than the reflector ab. Alſo a very ſmall hole muſt be made at v, to let the rays paſs thro' to the eye, which muſt be no bigger than the breadth of a pencil of rays at v.

117. Since ro, rE, rI are continual proportionals, I will be the focus of rays flowing from r; whence the image of the reflector ab will be painted diſtinct at I; and therefore a hole is to made in the plate at I, juſt the bigneſs of that image, for the rays to paſs thro' to the eye. If that hole be any bigger than the picture, the light coming by the ſide of the reflector ab, will go thro' the hole, and make the picture confuſed. And if the hole be leſs than the picture, rays from the extremities of ab will not paſs thro', and ſo the whole cannot be repreſented. Therefore to find exactly the diameter of the eye hole; it will be (by Prop. XVI. B. III), as $rE : ab :: EI$: breadth of the image at I, or the diameter of the hole required.

If you hold a thin paper at I, inſtead of the plate, the image of ab will be painted on it, and will be viſible thro' the paper, being a black ſpot very diſtinct, ſurrounded with light; and thus one may find the bigneſs of the hole experimentally. And the like for two eye glaſſes.

Fig. 117. To ſee objects that are near hand diſtinctly; or for long-ſighted people, to ſee far diſtant objects; the little concave *ab*, muſt be moved a little from the large one; but a little nearer for ſhort-ſighted people. And all this is done by turning the end L of the rod LM. For if an object comes nearer, its image at *n* will come nearer *t*; and as *tn* grows leſs, *to* will grow greater; and *o* will come nearer the lens E. Therefore to reduce it back, the glaſs *ab* is to be ſet further off. But for ſhort-ſighted people, the focus *o* muſt be ſet nearer E, to make the rays diverge more; that is, *to* muſt be longer, and conſequently *tn* ſhorter, or *ab* brought nearer FG. And the contrary for long-ſighted people. Therefore for far diſtant objects, and ſhort-ſighted people, ſcrew right about. And unſcrew for near objects, and for old men.

The properties of this teleſcope.

1. It ſhews the objects erect; but not ſo bright as in a refracting teleſcope; becauſe metal reflects leſs light than glaſs tranſmits. For by obſervation, it appears that all reflecting metals loſe more than a third part of the rays.

2. The viſible area of an object is as the breadth of the eye glaſs. For (fig. 117.) if the image at *o*, and the eye glaſs E be increaſed, the image at *n* will alſo be increaſed, becauſe the angles of incidence and reflexion at *ab* are equal; and conſequently the viſible part of the object is increaſed.

3. The brightneſs of an object is greater when the aperture is greater. For then more light comes to the eye.

118. 4. The brightneſs of the image is leſs towards the extreme parts; becauſe the ſhadow of the ſmall ſpeculum falls on the outſide; but on the middle it only covers the hole. And this diminution is greater in long teleſcopes.

5. It is found by experience, that a concave reflector, whoſe focal diſtance is 9.6 inches, will bear an aperture of 2.3 inches; and requires a ſmall ſpeculum, whoſe focal diſtance is 1.5 inches, and breadth 0.6 inches; and two eye glaſſes, whoſe focal diſtances are 3.8 and 1.1 inches. Whence the other requiſites are, the breadth of the hole = 0.5 inches; diſtance of the little ſpeculum and the greater eye glaſs = 14.2 inches; diſtance of the eye glaſſes 2.4 inches. The magnifying power of this is 60. Fig. 118.

6. A teleſcope of this ſort is, far ſhorter than a dioptric teleſcope that has the ſame magnifying power. For one of theſe reflectors 16 inches long, is found to magnify as much as a dioptric one of 15 or 16 feet long.

Mr. Caſſegrain's *teleſcope.*

ABCD is a tube open at the end AB; FG a concave ſpeculum whoſe focus is *n*, and in the middle of it, is a round hole for the reflected rays to paſs thro'; *arb* a ſmall convex ſpeculum whoſe focus is *t*; this is placed between *n* and FG, ſo that the focus *n* of the great concave, and the place of the image *o*, may be conjugate foci, or that *tn*, *tr*, *to*, may be continual proportionals. HI is a ſhort braſs tube, ſcrewing into the end of the larger, having a convex lens E in it, ſo placed, that E*o*, EI may be each equal to its focal diſtance. At I is a ſmall hole for the rays to come thro' to the eye, which is ſomething leſs than in Gregory's. 119.

Now when the open end is ſet towards the object, parallel rays coming from the object are reflected by the great concave FG, to the focus *n*, and there would make the image of it inverted (by Prop. XVIII. B. II.); but before they come at *n*, they are reflected by the little convex *ab*, Fig. 119. to the conjugate focus *o*, which being on the other ſide of the reflector, will (by the ſame Prop.) make there the ſecond image, erect in reſpect of that at *n*, but inverted in reſpect of the object. Then *o* being the focus of the lens E, the rays flowing from *o*, and being refracted at E, will emerge parallel to the eye at I; where the object will appear inverted; but may be made erect, by cauſing the focus *o* to fall without the tube HI, and applying a concave lens, as in Galileo's teleſcope.

To move the ſmall convex back and forward to ſuit all eyes and objects, there is the ſame contrivance as in the Gregorian teleſcope, by the rod LK. And inſtead of one eye glaſs two may be uſed, which muſt be placed as in the Gregorian. At *o* an eye ſtop is to be placed as uſual, to keep off ſtraggling rays.

The tract of a ray RG coming from the upper part of the object, will be RG*asw*I; where the part G*a* tends to *x*.

The properties of this teleſcope.

1. It ſhews the object inverted; and therefore is more proper for the night, where the inverſion of the objects is not regarded.

2. It is ſhorter than Gregory's teleſcope, by twice the focal diſtance of the leſſer ſpeculum; and has the ſame magnifying power, or rather more, and ſhews the object as bright.

3. The viſible area is as the breadth of the eye glaſs. For increaſing the eye glaſs, more rays will come from the outſides of the object.

4. The greater the aperture is, the brighter the object will appear. For then more light is brought to the ſame parts of the retina. This will alſo appear by ſtraitening the aperture, by putting a paper before it, with a hole in it.

5. The brightneſs of the object is ſomething leſs towards the extreme parts, occaſioned by the ſhadow of the ſmall ſpeculum.

The Newtonian teleſcope.

Fig. 120.

ABCD is a tube open at the end AB; FG a concave reflector, whoſe focus is *n*. *arb* a plain metal reflector, ſet to an angle of 45 degrees with the axis *rz* of the concave. Or inſtead of metal, you may have a glaſs priſm *abm*, the ſide *bm* being parallel, and *am* perpendicular to the axis *rz*, and *am* equal to *bm*, and the ſide *ab* muſt not be quickſilvered over; for it will reflect all the light without it. And whether this reflector be metal or glaſs, it muſt be an oval, whoſe length *ab* muſt be to its breadth, as $\surd 2$ to 1; becauſe it ſtands obliquely. This reflector is cemented to an arm L, which is fixed in the ſide of the tube, ſo that the axis of the ſpeculum may paſs thro' the middle of *ab*. Let *ro* be made equal to *rn*. And let *o* be the focus of the glaſs lens *e*, placed in the ſide of the tube; and a thin plate laid over the glaſs, with a ſmall hole in it; or rather a ſmall tube going from thence, of the length of *eo*, or rather more, as EI; at the end of which I is a ſmall hole for the rays to paſs thro' to the eye. The reflector *ab* muſt not be made too great, and the reflector FG may either be made of metal, or of glaſs, according to Prob. V.

The tube AD being turned with its open end AB towards the object; parallel rays coming from all the points of it, will be reflected from the concave FG to the focus *n*, where would be made the image of it inverted. But by the interpoſition of the ſmall reflector *ab*, they are hindered from coming to *n*, and are reflected thereby to the focus *o*, where they form the ſecond image equal to that at *n*. Laſtly, rays proceeding from this image, which is alſo in the focus of the lens *e*, will be refracted by the lens, and emerge parallel to the eye placed at I. Therefore if the face be ſet towards the

Fig. 120. the open end AB, and the eye applied at I, the object will appear inverted. Or if you ſet your face towards the other end CD, it will appear erect to the eye at I; but it is troubleſome to ſtoop down, except the teleſcope be nearly in a horizontal poſition.

The image may be made erect, by making the ſides *am* and *bm* of the glaſs priſm not plain but convex, that the rays may croſs, as well before they come at it, as afterwards between it and the eye glaſs. Or it may be erected, by projecting the focus *o*, without the tube, and applying a concave at *e*, of a proper focus.

The tract of an oblique ray RZ coming from the upper part of the object, is RZ*asw*I, where the part Z*a* tends to *x*, but is reflected in direction *as* by the plane *ab*.

The inſide of this tube, and indeed of all teleſcopes, muſt be made very black, to abſorb the rays of light that fall upon them.

All theſe ſorts of reflecting teleſcopes are ſet upon a ſtand with three feet; and are moveable in it every way, ſo that they may be directed to any object; which done, they are fixt there with a ſcrew, which keeps them ſteady in that poſition.

In ſuch of theſe teleſcopes as magnify much, there is a ſmall teleſcope fixt to the tube, by the help of which, any object may be found out, by directing it to the ſame object.

The properties of this teleſcope.

1. It will bear a greater aperture and eye glaſs than either of the others; and conſequently will magnify more by about a fifth part. It has likewiſe other advantages, for leſs light will be loſt by reflexion from the oval plane, than from a ſpherical reflector. And will alſo reflect the light more truly. In the other teleſcopes, the diſtance of the eye

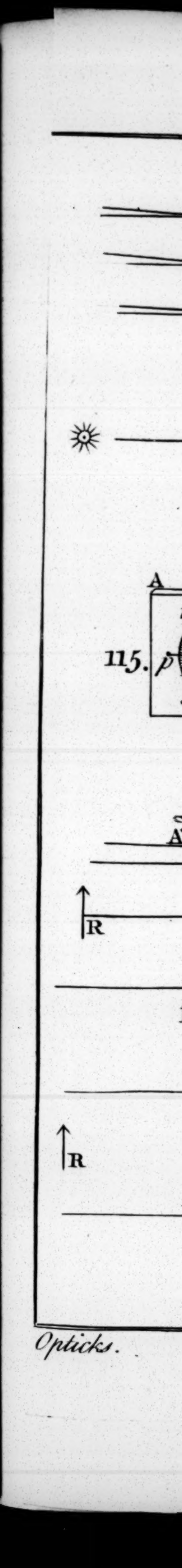
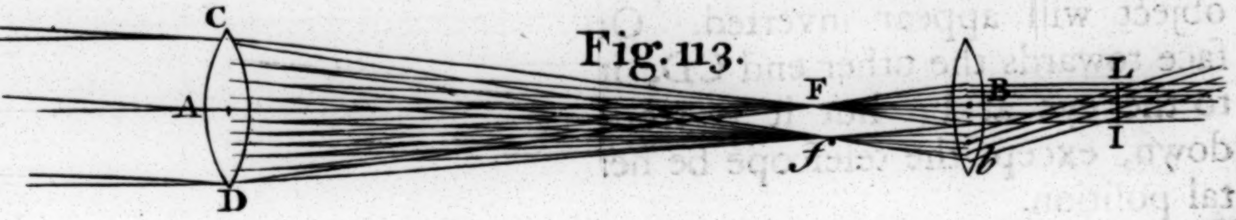


Fig. 113.



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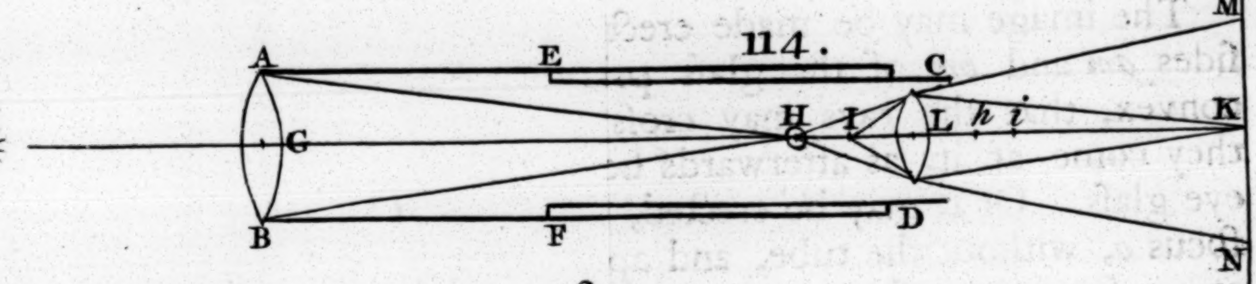

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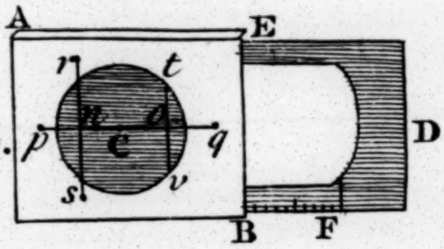

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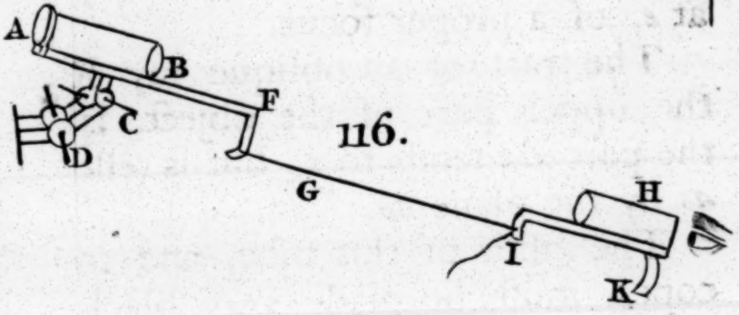

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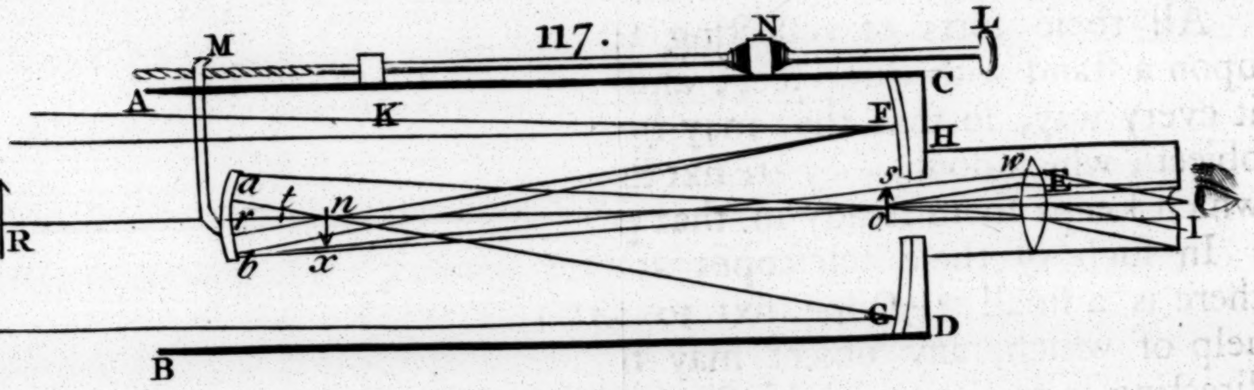

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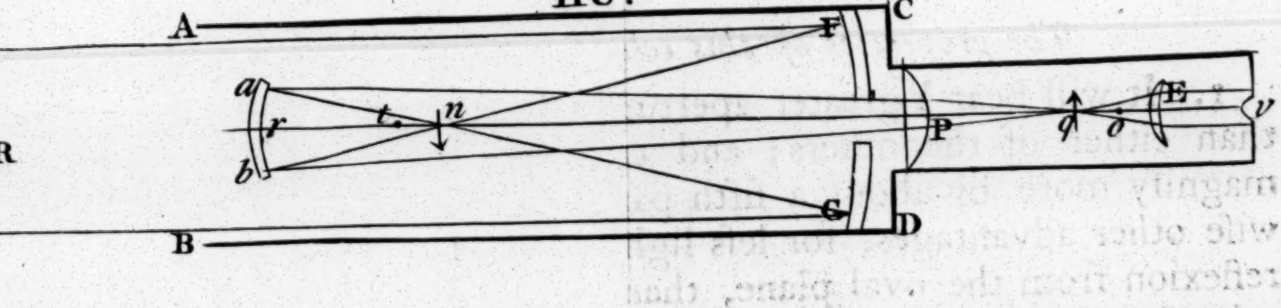

Pl. X. *pa.* 202.

B. I

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Fig. 120. eye glaſs and little reflector is greater, which augments the errors of refraction. The errors of the object metal will be more augmented by reflexion from the ſmall convex, which therefore requires a more exact figure; and hence it will not bear ſo large an aperture; alſo being convex it magnifies too much, and therefore will repreſent the objects obſcure, dark, and confuſed.

2. The object in this teleſcope appears inverted, and therefore is fitter for the night: beſides, it appears in a quite different place, which is not ſo natural, as when it is ſeen in its true place, by looking towards it thro' the teleſcope. And this makes it hard to find.

3. The viſible area is greater, the broader the eye glaſs is. And the object is brighter, the larger the aperture is.

4. This and other ſorts of reflecting teleſcopes, are ſhorter than dioptric ones of the ſame magnifying power. Becauſe the images made by refraction, are confuſed and imperfect, by reaſon of the different refrangibility of the rays, which has no effect in reflections. And conſequently theſe images will not bear to be ſo much magnified by refractors.

5. There is more care required in giving a true figure to a reflecting ſurface than to a refracting one. For an error in the figure of a reflector will produce almoſt ſix times as great an irregularity in the image, as it will do by the refractor.

PROB. XVII.

To find the magnifying power of a reflecting teleſtope; alſo the indiſtinctneſs and brightneſs, and viſual angle.

121. Let rV be the axis of the teleſcope, and let an object appear to the naked eye under the angle

RV*r*

Fig. 121. RVr or RVn, n being the focus of FG; and let nx be the firſt image made by reflexion from FG; and os the ſecond image made by reflexion from ab. Then drawing oE to the center of the eye glaſs, the angle sEo (by Prop. XVII. B. III.) is the angle which the object appears under thro' the teleſcope; and therefore it is magnified in the ratio of Rvr to oEs. But the angles of incidence and reflection being equal; we have $RVr = nVx$, and $nrx = ors$.

Now the ratio of RVr or nVx to oEs, is compounded of (RVr or) nVx to nrx, and of (nrx or) ors to oEs; that is, of rn to nV, and oE to or; or of $rn \times oE$ to $nV \times or$. And therefore the magnifying power is $\frac{nV \times or}{rn \times oE}$. But ſince t is the focus of ab, and (by conſtruction of the teleſcope) $tn : tr :: tr : to$, therefore $nr : tr :: or : to$, and $\frac{or}{rn} = \frac{to}{tr}$, and therefore the magnifying power is $\frac{nV \times to}{oE \times tr}$. This is in the Gregorian and Caſſegrain's teleſcopes. But in the Newtonian one, where the images nx and os are equal, tr and to have nothing to do; and its magnifying power is $\frac{nV}{oE}$ or (fig. 120) $\frac{nZ}{oE}$.

For the indiſtinctneſs.

The confuſion or indiſtinctneſs of the image in the focus of the reflector, is as the circle of aberration; and (by Cor. 1. Prop. XXIV. B. II.) the diameter of the circle of aberration is as $\frac{B^3}{4RR}$, that is, as the cube of the aperture directly, and the ſquare of the focal diſtance inverſly. And the apparent magnitude of that diameter ſeen thro' the eye glaſs, is as the diameter itſelf directly, and the focal diſtance of the eye glaſs inverſly. And the indiſtinctneſs in the eye is as the apparent area of that circle, that is as the 6th power of the aperture directly, and the 4th power of the focal diſtance of the great reflector inverſly, and alſo as the ſquare of the focal diſtance of the eye glaſs inverſly. Therefore if A be the aperture, and F the focal diſtance of the large reflector, g the focal diſtance of the eye glaſs; then the indiſtinctneſs is as $\frac{A^6}{F^4gg}$. Fig. 121.

For the brightneſs.

If the area of the image in the eye be given, the brightneſs will be as the quantity of light, that is as the ſquare of the aperture. And if the light, or the aperture be given, the brightneſs will be as the area inverſly. Therefore in general the brightneſs is as the ſquare of the aperture directly, and the area inverſly, but the area is as the ſquare of the magnifying power; therefore the brightneſs is as the ſquare of the aperture directly, and the ſquare of the magnifying power reciprocally. Therefore if M be the magnifying power, the brightneſs is as $\frac{A^2}{M^2}$.

Cor. 1. *If* F, f, *be the focal diſtances of the great and ſmall reflector*; g = *focal diſtance of the eye glaſs*, *to* = d. *Then in the Gregorian and Caſſagrain's teleſcope, the magnifying power is* $\frac{Fd}{fg}$. *And in the Newtonian, it is* $\frac{F}{g}$.

Cor. 2. *An object appears equally diſtinct in reflecting teleſcopes of the ſame ſort; when the focal diſtances of*

Fig. 121. *of the eye glaſſes, are as the cubes of the apertures of the great reflectors, divided by the ſquares of their focal diſtances.*

For if $\frac{A^6}{F^4g^2}$ be given; A^6 is as F^4g^2, and A^3 as $F^2 g$, whence g is as $\frac{A^3}{F^2}$.

Cor. 3. *In reflecting teleſcopes of the ſame ſort; an object appears equally bright, when the aperture is as the magnifying power.*

For if $\frac{A^2}{M^2}$ be a given quantity, then A^2 is as M^2, and A is as M.

Cor. 4. *In ſeveral reflecting teleſcopes of the ſame ſort, an object will appear equally bright and diſtinct; when their apertures and magnifying powers are each as the fourth roots of the cubes of the focal diſtances of the great concaves; and the focal diſtances of the eye glaſſes as the fourth roots of theſe focal diſtances.*

For in the Newtonian teleſcope M is as $\frac{F}{g}$; and ſince A is as M or $\frac{F}{g}$, therefore Ag is as F, and g as $\frac{F}{A}$, but g is alſo as $\frac{A^3}{F^2}$; therefore $\frac{F}{A}$ is as $\frac{A^3}{F^2}$, and F^3 as A^4, and A as $\sqrt[4]{F^3}$; and therefore M is as $\sqrt[4]{F^3}$. Whence g is as $\frac{F}{\sqrt[4]{F^3}}$ or as $\sqrt[4]{F}$

And in the Gregorian and Caſſegrain's teleſcopes, we have M as $\frac{Fd}{fg}$, wherein if we ſuppoſe $\frac{d}{f}$ or (which is the ſame thing) $\frac{tr}{tn}$ a given ratio, which ſuits them very well; then M will be as $\frac{F}{g}$, and all things will follow as before in the Newtonian teleſcope. Fig. 121.

Cor. 5. *In reflecting teleſcopes the viſual angle is as the breadth of the eye glaſs, divided by its diſtance from the ſmall reflector.*

For the angle of reflection at r being equal to the angle of incidence, the ray rs will miſs the lens E, if it is not broad enough. Therefore diminiſhing the lens, diminiſhes the angle srV and Vrx; and alſo rVx, and rVR the viſual angle; and $< sro$ is as $\frac{so}{or}$ = breadth of the eye glaſs, divided by Er.

Cor. 6. *If* N *be the angle, which the eye glaſs ſubtends at the little reflector. Then* $\frac{Nf}{F}$ *is the angle the teleſcope takes in, nearly; ſuppoſing the hole as big as the eye glaſs. But in Newton's, the angle taken in, is that which the eye glaſs (placed in the focus) ſubtends at the great concave.*

For ſuppoſing the ray rs to touch the edge of the eye glaſs, then the angle N = ors or xrn, is to be diminiſhed in the ratio of rn to nV to get the angle nVx or RVn, that is nearly in the ratio of rt to nV, or f to F. And in Newton's, nx ſubtends the angle nZx = RZn (fig. 120.) The angle N is the linear view; and that diminiſhed in the ratio of f to F nearly, is the real angle, the part ſeen of the object, appears under, to the naked eye.

Cor. 7. *In ſimilar reflecting teleſcopes, the magnifying power, diſtinctneſs and viſual angle, are the ſame. And the brightneſs as the ſquare of the length of the teleſcope.*

For F, f, g, are in given ratios to one another; and the magnifying power is as $\frac{Fd}{fg}$ or $\frac{F}{g}$, which

is

Fig. 121. is a given ratio. And the diſtinctneſs (or its ſquare root) is as $\frac{A^3}{F^2 g}$, and this is a given ratio. Alſo the viſual angle is as $\frac{b}{F+f}$ or as $\frac{b}{F}$ (putting $b =$ breadth of the eye glaſs), which is alſo a given ratio. And laſtly, the brightneſt is as $\frac{A^2}{M^2}$ or as A^2 (becauſe M is given) that is, as F^2.

Cor. 8. *From what has been laid down concerning the nature and conſtruction of theſe teleſcopes, it will not be difficult to conſtruct one of any length. I ſhall here inſert ſome tables conſtructed by the preceding rules.*

The Gregorian Teleſcope.

F		f	A	a	g	M
f.	in.	inches	inches	inches	inches	
0	6	1.14	1.61	0.32	1.25	25
0	9	1.46	2.23	0.38	1.53	36
1	0	1.76	2.70	0.42	1.73	47
1	6	2.32	3.62	0.51	2.04	68
2	0	2.67	4.52	0.56	2.31	93
3	0	3.43	6.26	0.65	2.56	148

Mr. Caſſegrain's.

F		f	A	a	g	M
f.	in.	inches	inches	inches	inches	
1	6	2.17	3.89	0.42	1.67	90
3	0	3.57	6.57	0.59	2.34	155
5	0	5.17	9.61	0.76	3.03	230

Fig. 121.

Sir J. Newton's.

F	A	g	M
feet	inches	inches	
$\frac{1}{2}$	0.86	0.16	36
1	1.44	0.20	60
2	2.45	0.23	102
3	3.31	0.26	138
4	4.10	0.28	17
5	4.85	0.29	202
6	5.57	0.31	232
8	6.89	0.33	287
10	8.16	0.35	340
12	9.36	0.36	390

SCHOLIUM I.

The breadth of the ſpeculum *ab* is not determined by the foregoing calculations, and therefore is left to the judgment of the artiſt. But it muſt be conſidered, that if it is too broad, it will intercept too many of the beſt rays, from coming to the large concave FG; and if it be two ſmall, many of the rays reflected from FG, will paſs by it. Therefore to avoid both inconveniences, it will be proper to make its breadth to that of FG, as their focal diſtances, or rather as *rn* to V*n*. For then it will juſt reflect the whole pencil of direct rays that falls upon FG, (ſee fig. 118.)

Likewiſe $\frac{d}{f}$ was aſſumed a given ratio; but as it is unlimited, it may be taken otherwiſe; and then this will cauſe a variation in all the other quantities, and their proportions to one another. And of this experience will be the beſt guide.

In the tables before laid down, it is ſuppoſed that the focus *o* falls at or about V, and on that ſup-

Fig. ſuppoſition the magnifying powers are calculated;
121. which would be greater if *o* falls beyond it.

Inſtead of a ſmall concave or convex reflector *ab*, a plane reflector may be uſed. But then to have the focus *o* to fall at V, it muſt be placed in the middle of *n*V; and then its breadth muſt be half of FG, or elſe it cannot reflect all the direct rays falling on FG, and reflected by it. Such a plane reflector would ſtop a quarter of the direct pencil of rays, and the beſt of them too. And beſides, the image at *o* would be no bigger than that at *n*; and therefore this conſtruction will not anſwer the end.

Very long teleſcopes ſhould not be uſed to magnify terreſtrial objects. For they equally magnify the particles of the atmoſphere, and all groſs vapours ariſing in it. And thro' ſuch teleſcopes the air appears in a perpetual tremor; and the more ſo, the more the teleſcope magnifies. And this trembling motion of the atmoſphere makes every object appear confuſed and indiſtinct. The only remedy is a ſerene and quiet air.

If it was not for the different refrangibility of the rays, the errors in refracting teleſcopes, would only be ſuch as are cauſed by the ſphericalneſs of the figure of the glaſſes; and then the dimenſions of the ſeveral parts would be proportioned by the ſame rules, as in reflecting teleſcopes. But as experience ſhews the contrary, it is plain, that the aberration ariſing from the ſpherical figure, is extremely ſmall, in compariſon of that proceeding from the different refrangibility.

I ſhall here add the algebraic expreſſions for the ſeveral parts of Gregory's teleſcope, ſuch as have been found to anſwer experience.

Let focal diſtance of the great concave = F.
its breadth — — — = A.
magnifying power — — = M.

tan. angle the hole in the concave ſubtends at the eye glaſs $\Big\}$ $= n$. Fig. 121.

$$\frac{nF}{M} = c,\ \sqrt{Ac + cc} = R.$$

then, $c =$ breadth of the image at n, of ſo much of the object as the eye takes in; ſuppoſing the eye glaſs as big as the hole.

Breadth of the hole in the concave, or of the ſmall ſpeculum (a) $\Big\}$ $= c + R$.

focal diſtance of the ſmall ſpeculum $(f) = \frac{F \times \overline{c + R}}{A + c + 2R}$.

diſtance of the ſpeculums $(Vr) = F + \frac{F\sqrt{c}}{\sqrt{A + c}} = l$.

focal diſtance of the eye glaſs $(g = Eo) = \frac{c + R}{n}$.

diſtance of the eye (EI) $= \frac{l + g}{l} g = h$.

breadth of the hole in the plate at the eye $= \frac{h \times \overline{c + R}}{l + g}$.

Note, day teleſcopes require an eye glaſs of twice the focus of night teleſcopes; and magnifies but half as much.

SCHOLIUM II.

Beſides the teleſcopes deſcribed before; there is alſo one called a *binocular or double teleſcope.* This is no more than two equal teleſcopes ſet in a frame, parallel to one another; and theſe may be ſet at a proper diſtance from one another, by the help of ſcrews; and that diſtance is to be the ſame, as the diſtance of the two pupils of the eyes. When that is adjuſted, a perſon is to look thro' them both at once; thro' one with each eye, to any object; which will then be ſeen by both eyes, and appear far brighter than thro' a ſingle teleſcope.

Fig. All teleſcopes in general repreſent all terreſtrial objects to be nearer hand, but not larger; and this nearneſs, vicinity or ſeeming approach of the object, is as the magnifying power of the teleſcope. Thus looking at a man 100 yards off, with a teleſcope that magnifies 100 times; the man will appear no bigger, but will ſeem only to be a yard off; and the like of other objects ſituated on the earth.

PROB. XVIII.

To find the magnifying power of a teleſcope, by experiment.

Make two equal circles of paper of an inch diameter or more, and fix one of them upon a wall 100 or 200 yards diſtant; and the other at a ſmall diſtance, in a line with the firſt. Then look at the further circle thro' the teleſcope with one eye, and at the near circle with the other eye naked. Move the nearer circle (or elſe the teleſcope) back and forward, till the two circles appear equal, or coincide. Then meaſure the two diſtances, from the eye glaſs of the teleſcope to the two cirles; then divide the greater diſtance by the leſſer, and you have the magnifying power of the teleſcope.

Or thus,

If you be in a room where there is a caſement or ſaſh window; cauſe a ſtaff, or ſuch a thing of two or three feet long, to be ſet upright at a good diſtance from the houſe. Open the window, and place yourſelf at a convenient diſtance therefrom, ſo that you may be in a line with the ſtaff, and the ſide or frame of the window. Then looking thro' the teleſcope with one eye, to the ſtaff, mark the extremities of the ſtaff where they appear on the window frame. Alſo with the other eye naked, (or with the ſame eye afterwards, placed where Fig. the eye glaſs of the teleſcope was) look at the ſtaff, and mark the extremities of it alſo on the window frame. Meaſure how oft this laſt diſtance is contained in the firſt, and that is the magnifying power.

Inſtead of a ſtaff, an accidental object may preſent itſelf, which will do as well; as a chimney top, a window or door of a houſe, a tree, &c. By trying thus ſeveral times to find if your marks be rightly ſet, you may do the thing pretty exactly.

In a refracting teleſcope, when you have a large object to look at, you may do the ſame thing, by looking the contrary way thro' the teleſcope to diminiſh it; and making marks as before; and likewiſe looking with the naked eye, and making marks likewiſe; for theſe will give the ſame reſult. Inſtead of marks, you may ſtick pins into the window frame.

PROB. XIX.

To make a ſingle microſcope.

A *microſcope* is an optical inſtrument, made of 122. one or more glaſſes, or ſometimes with a reflecting metal; to view very ſmall objects.

To make a ſingle one, take a very ſmall lens L, fix it in a frame to take hold on with the fingers; and place any ſmall object PR in the focus of it; and applying the eye cloſe to it on the other ſide, at O, and looking thro' the lens; and it will view the object diſtinctly and magnified. Or if the focus is not known, move the object to and fro', till it appears moſt diſtinctly; and then it is in the focus; and therefore the rays will be refracted parallel to the eye.

In this microſcope, put D = the leaſt diſtance, that the naked eye can ſee diſtinctly; then the mag-

Fig. 122. nifying power of the microſcope is $\frac{D}{QL}$. And it is ſeen direct.

For the rays from P and R enter the eye under the angle PLR, but to the naked eye the object appears under the angle which PR ſubtends at the diſtance D. And theſe angles are reciprocally as the diſtances D and QL. And therefore the apparent magnitudes are in that ratio.

If the lens L was taken away, and a plate with a ſmall hole in it put in its place; the object PR will appear under the ſame angle PLR as before.

Otherwiſe thus.

123. Since lenſes cannot be formed ſmall enough to view very minute objects; there is a neceſſity of making uſe of very ſmall ſpheres for that purpoſe. Let ABD be a ſmall ſphere, C its center, PQR an object, placed in the focus Q, where $AQ = \frac{1}{2}AC$. Then applying the eye cloſe behind the ſphere, it will ſee the object magnified. For the rays flowing from the focus Q, and falling on the ſphere, will emerge parallel to the eye, and the object will appear diſtinct, and erect.

If D be the leaſt diſtance that the naked eye can ſee diſtinctly; then the magnifying power will be $\frac{D}{CQ}$.

For drawing PCG, RCF thro' the center C, the rays from P and R come to the eye under the angle FCG or PCR. And that angle is to the angle which the object appears under at the diſtance D, as that diſtance D to CQ; and therefore that is the ratio of the magnifying power.

Whence this ſphere has the ſame magnifying power as a lens whoſe focal diſtance is CQ. But theſe ſpheres are inferior to lenſes; becauſe in the lens the object is three times as far diſtant from it, as it is from the ſphere; for which reaſon the ſphere hides almoſt all the light from the object. Fig. 123.

Theſe ſpheres are fixt between two braſs plates, or rather one plate doubled in the middle; and having a ſmall hole drilled through it, to take up about thirty degrees of the ſphere; and in this hole the ſphere is lodged.

Otherwiſe thus.

AB is a tube, having a ſmall lens or ſphere fixt in the end A, and the end B muſt be covered with a very thin paper. P is the object, which muſt be placed a little further off than the focus; ſo that the other focus (conjugate to P) may fall juſt on the paper at B; which is eaſily known by moving P back and forward, till the image at B be diſtinct, where it will appear magnified, in the ratio of the length of the tube to the focal diſtance of the glaſs ſphere or lens, and inverted. The object P ought to be illuminated. 124.

A microſcope may be made, by placing two plano convex glaſſes in a frame, to touch at their convex ſurfaces. This will ſhew the objects flat and not crooked.

Cor. 1. *In ſingle microſcopes, the object will be equally diſtinct, when the apertures are as the focal diſtances.*

For then they are ſimilar parts of the whole ſpheres, and conſequently there is the ſame ſeparation of the rays in both, upon which the indiſtinctneſs depends.

Cor. 2. *The magnifying power of a ſingle lens is reciprocally as the focal diſtance; or it is* $\frac{D}{QL}$.

For D the diſtance to ſee diſtinctly, is given.

Fig. 122. Cor. 3. *An object would appear as great to the naked eye as thro' a ſingle microſcope, if it was placed at the focal diſtance from the eye, and the eye could ſee diſtinctly at that diſtance.*

For then $D = QL$, and the magnifying power $\frac{D}{QL}$ is 1; and it would appear under the ſame angle to the naked eye as thro' the microſcope.

Cor. 4. *The brightneſs of an object ſeen through a ſingle microſcope is as the ſquare of the aperture, divided by the ſquare of the focal diſtance.*

For the focal diſtance and aperture being ſuppoſed in a given ratio, the angle which the glaſs ſubtends at the focus is given, and conſequently the quantity of light contained in that angle (or in a cone deſcribed by its revolution) will be given. But when a given quantity of light enters the eye, the brightneſs of the image will be reciprocally as its bigneſs, that is reciprocally as the ſquare of the magnifying power, or reciprocally as the ſquare of the focal diſtance. Now increaſe the aperture, and the light will be increaſed in the ratio of the ſquare of the diameter of the aperture.

Cor. 5. *The more convex the lens is, the greater is the image; but then it is alſo the more obſcure. And therefore we cannot magnify as far as we pleaſe, unleſs more light be borrowed to illuminate the object; and that would fail at laſt.*

For the power of the microſcope would be limited by the breadth of the pencils, that enter the pupil, and theſe are equal to the breadth of the aperture; and if that be leſs than $\frac{1}{5}$ or $\frac{1}{6}$ of a line; the edges of the object will begin to appear indiſtinct. And to preſerve the ſame diſtinctneſs, the aperture cannot be increaſed, but muſt be in a given ratio to the focal diſtance.

Fig.

SCHOLIUM.

There is a way to make water microſcopes, 124. which is thus; take a thin braſs plate, and dril a hole in it, about the 30th part of an inch. Then take a little water upon the point of a needle or pin, and fill the hole with it, and this water will form a double convex lens or ſphere. Then a ſmall object being placed upon a thin glaſs, fixed in the other end of the braſs turned up, and drawn into the focus by a ſcrew. Then the object will appear magnify'd thro' the drop of water, more or leſs according to the ſmallneſs of the hole. But this ſphere of water will exhale, and vaniſh away in a little time.

If an object be illuminated by ſome homogeneal colour in a dark room, the microſcope will bear a deeper charge and a larger aperture, with diſtinctneſs.

To make the little glaſs ſpheres for microſcopes. Take ſome ſmall pieces of broken glaſs, and hold them in the blue flame of a candle, till they be red hot, and run into globes, or rather in a lamp made with rectified ſpirit of wine, with a wick made of ſilver wire laid cloſe together. Any ſmall piece of glaſs taken upon the end of a ſilver needle wetted with ſpittle, and this held in the flame till it is melted, and becomes round, then it muſt be immediately taken out and cooled. Or take a clear piece of glaſs and heat it in the lamp, and draw it out into a long thread; then holding the end of the thread in the flame, till it run into a round globule hanging at the end of the thread; it muſt then be taken out. Theſe will magnify vaſtly if they are as big as a grain of muſtard ſeed, or of a ſmall pin head. The chief uſe of theſe globules, is to look at tranſparent particles of matter;

ter; and the leaſt globules have the greateſt power of magnifying.

Drops of liquors which are to be viewed, muſt be ſpread upon a thin glaſs, contrived to ſlide every way, that all the parts of it may be brought ſucceſſively to the focus.

By help of theſe inſtruments one may ſee the circulation of the blood in animals; the animalcula that ſwim in drops of water impregnated with ginger, pepper, and ſuch like things; alſo in vinegar. Their forms are various, and their motion odd and ſurpriſing; theſe in vinegar are like eels, and ſwim like them; and generate their own kind. It is likely theſe animals are drawn out of the air by the ſmell of the infuſion in which they are found. For none appears in a veſſel that is cloſe covered. Milk appears to conſiſt of tranſparent globules, ſwimming in a tranſparent liquor, but of a different degree of refraction. The animalcula that ſwim in water are of a globular form, but tranſparent; they have two motions, one progreſſive, and the other a rotary one; they are exceedingly ſmall, 30,000 of them being but equal to a grain of ſand. In rivers after rain there are plenty. Rain water has many, and ſnow water more. The dew and the air is full of them.

PROB. XX.

To make a compound Microſcope with two or more convex lenſes.

125. Let the microſcope be made with the two lenſes A and B; A the object glaſs, which muſt be a very ſmall one, and very convex; F its focus. B the eye glaſs, being larger than the other. PR is the object, which muſt be placed a little beyond the focus F, of the glaſs A. Make QF, QA,

Qq continual proportionals; and make q the focus of the glaſs B. Alſo make Aq AB, AO continual proportionals, and O is the place of the eye, where looking thro' the glaſs B, it will ſee the object at q inverted. Theſe glaſſes are incloſed in a tube, which may be brought nearer, or ſet further from the object PR, as there is occaſion, by ſliding within another tube. Generally there are ſeveral ſizes of the lens A, fixt in little cells or buttons, which ſcrew upon the end of the tube, as one has occaſion to uſe a greater or leſſer magnifier. The objects are laid on thin pieces of glaſs or talk, to which they cleave, and theſe are fixed in ſliders, to be uſed occaſionally. It is beſt to ſet the tube upright, and look down perpendicularly upon the object, thro' a ſmall aperture made at O. There is a concave or convex glaſs in moſt of theſe machines, to throw light upon the object PR. Fig. 125.

Since QF, QA, Qq, are in continued proportion, Q and q, are conjugate foci; (by Prop. XXI. B. III.) and *pr* is the image of PR. And ſince q is the focus of B; the rays flowing from *pr*, and refracted by B, will emerge parallel to the eye at B; where it will ſee the image *pr* diſtinct, and in a contrary poſition to PR. Likewiſe ſince A*q*, AB, AO are continual proportionals, A and O are conjugate foci; and (by Prob. III.) therefore O is the place where all the pencils meet, and conſequently is the place of the eye, to ſee the moſt of the object.

If D = the leaſt diſtance the naked eye can ſee diſtinctly, then the magnifying power is $\frac{D \times qA}{qB \times QA}$.

For in the ſingle microſcope B (Prob. XIX.), the image *pr* is magnified in the ratio of D to *q*B. And by the lens A, the object PR is magnified (to *pr*), in the ratio of A*q* to AQ, (by Prop. XXXII.

Fig. 125 XXXII. B. III.) ; therefore the whole magnifying power is $\frac{D\times Aq}{qB\times AQ}$. Alſo ſee the Corollaries of Prob. II.

To ſuit the microſcope to ſhort-ſighted people, the glaſſes A and B muſt be placed a little nearer together, that the rays of each pencil may come a little diverging to the eye.

The lens B muſt be greater than A, otherwiſe many of the rays paſſing from the object through A, would miſs B ; therefore it muſt be broad to receive them ; and the axes of all the pencils will meet at O. This conſtruction is nearly the ſame as in the aſtronomical teleſcope ; only the object glaſs A is here leſs than the eye glaſs, which muſt be greater in the teleſcope.

In many of theſe ſorts of microſcopes, there are two eye glaſſes ; and the middle one is the largeſt; and is placed before the image *pr*, between *q* and A. By this means the microſcope receives more light. And theſe two eye glaſſes bring the rays ſooner to a focus, and gives a larger view of the object.

Cor. 1. *The magnifying power of a double microſcope* AB, *is* $\frac{D\times qA}{qB\times QA}$. *And is ſomething leſſer in a treble microſcope: which is ſhorter, where* D *is the diſtance to which the eye can ſee diſtinctly.*

Cor. 2. *The apparent brightneſs of an object* PR, *is as* $\frac{A^2\times qB^2}{qA^2}$; *putting* A *for the aperture of the object glaſs* A.

For the light falling upon A in the plane PAR, is as the angle it ſubtends at Q, that is, as $\frac{A}{AQ}$; and that cone of light is as $\frac{AA}{AQ^2}$; and that light is tranſmitted to the image pr, and ſo to the bottom of the eye; and its brightneſs there will be as the denſity, or reciprocally as the area of the image; that is, reciprocally as the magnifying power. Therefore the brightneſs is as the ſquare of $\frac{A}{AQ} \times \frac{Bq \times AQ}{D \times Aq}$; that is, becauſe D is given, as the ſquare of $\frac{A \times Bq}{Aq}$.

Fig. 125.

Cor. 3. *The viſual angle is as the breadth of the eye glaſs.*

For if the eye glaſs be contracted, ſome of the extreme rays will miſs it, and ſo the view is contracted.

Cor. 4. *It is found by experience, that a double microſcope performs well when the aperture of* A *is* $\frac{1}{10}$ *of an inch,* AF $= \frac{7}{10}$. $Aq = 7$, *and* $Bq = 2$. *And the magnifying power* $= 36$.

SCHOLIUM.

Altho' the aberration occaſioned by the figure of the glaſs is of no moment in teleſcopes, when compared with the aberration ariſing from the different refrangibility of the rays; yet in ſuch ſmall lenſes as are uſed for microſcopes, where the apertures are large, in reſpect to the focal diſtance; the aberrations by the figure become ſenſible, and often cauſe the image to be indiſtinct. For example, let there be a ſingle microſcope, whoſe lens is $\frac{1}{10}$ of an inch in breadth, and its focal diſtance near $\frac{4}{10}$. Then (by Cor. 6. Prop. XLI. B. III.) the aberration by the figure, will be to the aberration by the colours; as 232 to 256, almoſt equal. So that in ſmall lenſes, the indiſtinctneſs is cauſed by both ſorts of aberration, and therefore cannot well be brought under any rule.

A ſmall

Fig. 125. A ſmall aſtronomical teleſcope may be turned into a microſcope, by drawing the eye glaſs ſomething further from the object glaſs; and looking thro' it the contrary way, at the object placed near the end of it.

PROB. XXI.

To make a reflecting microſcope.

126. Let FG be a reflecting metalline concave, whoſe focus is P, BC a refracting lens for the eye glaſs, placed upon the ſame axis. O an object, placed near the focus O. Take PO, PV, PA continually proportional, and place the lens BC ſo, that A may be its focus. Then take VA, VE, VI, continually proportional, and I is the place of the eye, which thro' the eye glaſs BC, will ſee the object at A diſtinct, inverted and magnified.

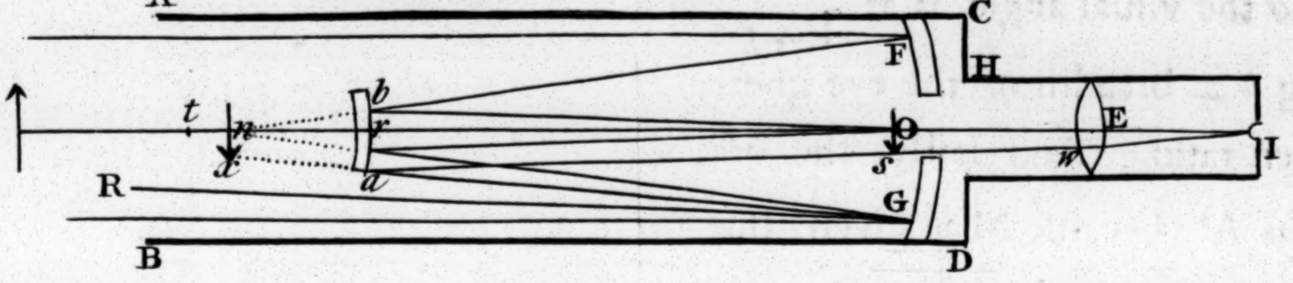
For O and A being conjugate foci, the object at O will be repreſented at A inverted. And A being the focus of BC, the rays flowing from A will emerge from BC parallel to the eye at I, and cauſe diſtinct viſion.

The eye glaſs E is incloſed in a tube KL, which ſlides within another KH, which is cloſe as far as DH, but open in the ſpace DFGH, to let the light in upon the object O. A ſtop muſt be ſet at A to cut off the ſtraggling rays. To illuminate the object, it will be proper to throw the light upon it, with a concave reflector.

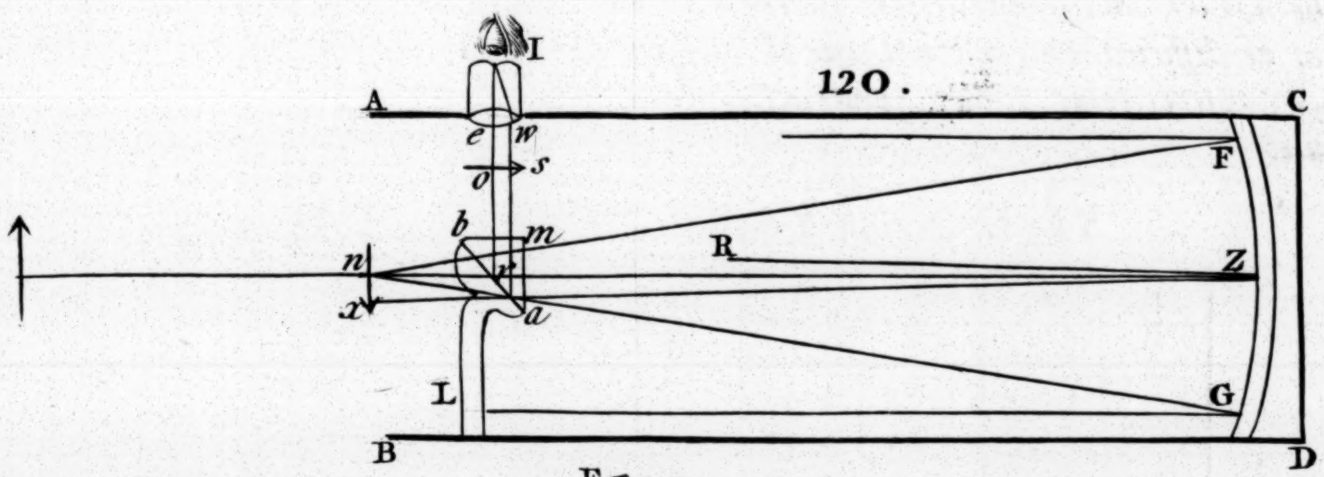
Cor. 1. *If* D *is the leaſt diſtance that the eye can ſee diſtinctly (as eight inches); then the magnifying power is* $\frac{VA \times D}{VO \times AE}$.

For

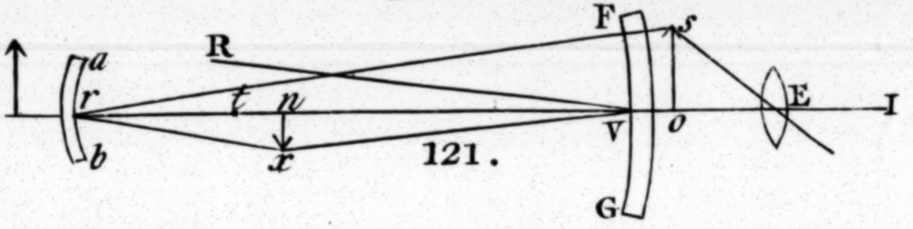
Fig. 119.



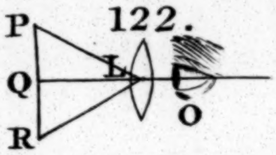
120.



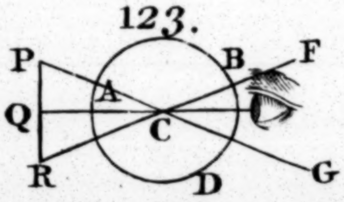
121.



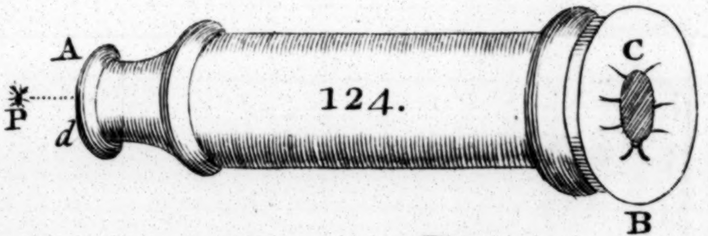
122.



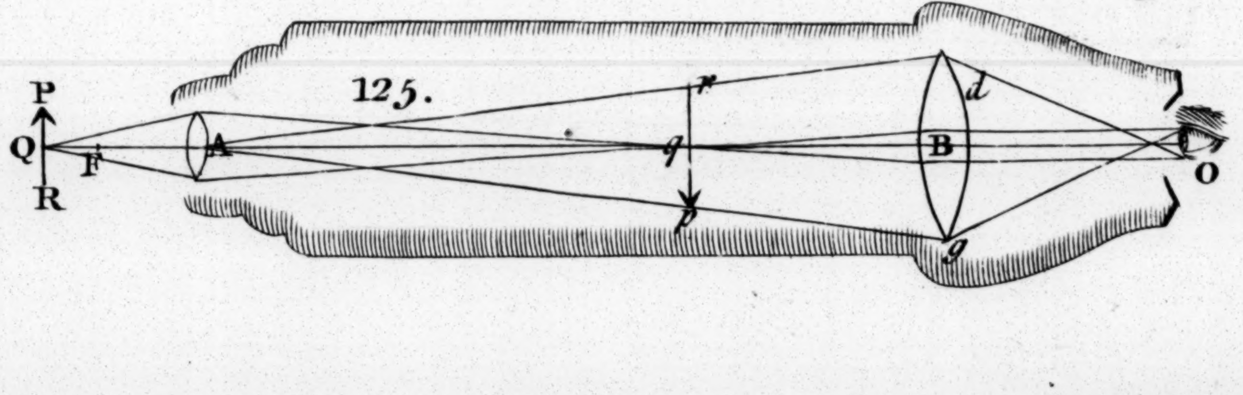
123.



124.



125.



Opticks.

Pl. XI. *pa.* 222.

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For the object is magnified at A, in the ratio $\frac{VA}{VO}$ (by Prop. XVI. B. II.) and this image at A s again magnified by the lens BC, in the ratio $\frac{D}{AE}$ (by Cor. 2. Prob. XIX.) Fig. 126.

Cor. 2. *The apparent brightneſs in ſeveral microſcopes, is as* $\frac{AA\times AE^2}{VA^2}$. *Where* A *is the aperture of* FG.

For the light falling on FG, is as the ſection of he cone FOG, or as $\frac{AA}{VO^2}$; and that light carried to the image at A, and thence to the eye, has the brightneſs diminiſhed in the ratio of the area of the image in the eye, or as the magnifying power. Therefore the brightneſs is as the ſquare of $\frac{A}{VO}\times\frac{VO\times AE}{VA\times D}$ or of $\frac{A\times AE}{VA\times D}$ or of $\frac{A\times AE}{VA}$.

Cor. 3. *It is found by experience, that a microſcope anſwers very well, that has the following dimenſions*; FG = 1 *inch*, VP = $\frac{7}{3}$, AE = $\frac{7}{3}$. PO = $\frac{1}{14}$ PV.

SCHOLIUM.

A reflecting microſcope may alſo be made after the manner of the Gregorian teleſcope, with two concave ſpeculums, the object being placed without the tube at a ſmall diſtance from the end of it. Or it may be made after the form of Mr. Caſſagrain's teleſcope, with a concave and a ſmall convex reflector; and the object to be placed between the principal focus and center of the large concave, and a little without the ſmall reflector; and then each of the reflectors muſt have a hole made thro' it, to let the rays paſs from the object.

PROB.

Fig.

PROB. XXII.

To make a ſolar microſcope.

The uſe of this inſtrument is to magnify tranſparent objects which are very ſmall; and to expoſe them to view by the help of the ſun's light thrown upon them, and caſting their image on the oppoſite wall of the room.

127. The conſtruction is this; CD is a microſcope glaſs, F its focus, its focal diſtance GF, which may be ¼ of an inch or ½ an inch. The glaſs CD is fixt in the tube EK, which ſlides within another tube HI about two inches diameter, and eight or ten inches long. O is a very ſmall object, placed on a ſlip of Muſcovy glaſs or talk, in a ſlider, which goes thro' the tube HI, where a ſlit is made on each ſide to receive it. By ſliding the tube EK containing the microſcope glaſs CD, within the other tube HI back and forward, the glaſs is ſet to any diſtance from the object O, which is to be placed ſomething beyond the focus F. The place of the object O, is adjuſted thus; let L be the oppoſite wall of the room; make OF, OG, OL continual proportionals; and O, L will be conjugate foci, by Prop. XXI. B. III.

To illuminate the object, a lens AB is fixt in the end of the tube HI, to collect the ſun beams, and throw them upon the object at O, which muſt be placed between the glaſs AB and its focus; for if the object was in the focus of CD it would be burnt or diſperſed by two much heat.

This inſtrument is only uſed in a dark room, to which purpoſe the end of the tube HI goes thro' a ball and ſocket, which fits into a hole in the window ſhut MN; then the tube being directed towards the ſun; the lens AB will caſt the ſun's rays upon the object at O, by which it is ſtrongly illuminated. Then rays flowing from it as a luminous body, will be refracted by the lens CD, and converge to L, where it will paint the image of that body. For O and L being conjugate foci, the image will appear diſtinct at L. And therefore the tube EK muſt be ſlid in or out, till the image at L appears diſtinct. Fig. 127.

There is a contrivance of placing a looking-glaſs without the hole of the window ſhut, which may be ſet at any angle. And by turning ſome ſcrews or pins, it is ſet to ſuch an angle as to reflect the ſun's rays horizontally thro' the tube; ſo that it need not be elevated to the ſun.

By the help of this inſtrument may be viewed the animalcula in pepper water, in vinegar, or any other liquor; and likewiſe any other ſmall animals that are tranſparent.

It is by the means of the ſun's light that the picture at L becomes viſible; for ſeeing it is very much magnified; the brightneſs would decreaſe in proportion to its magnitude, and it could not be ſeen at all with common light.

Any object O is magnified in proportion of GL to GO, or the diſtance of the image from the lens CD, to the diſtance of the object from it. And therefore the ſmaller the lens CD is, or the leſſer FO is, the more the object will be magnified.

PROB. XXIII.

To make a magic Lanthorn.

ABPQR is a tin lanthorn a foot diameter; or it may be a ſquare box of wood; HI a round tube four inches diameter, and eight inches long, fixt in the ſide of it. CK another tube four inches long, ſliding within the firſt, and having a 128.

Fig. 128. convex lens D of three inches focus, fixed in it. O an object whoſe image is to be projected on the oppoſite wall of the room at L. F is the focus of D, and O is to be placed beyond F, ſo that OF, OD, OL, be continual proportionals; and then L will be the place of the image. This inſtrument is only uſed in a dark room; and to illuminate the object, a candle or lamp is ſet at E, and a concave reflector GN placed on the oppoſite ſide; ſo that its vertex may be in the ſame line with E, O, F, D; and that E and O may be conjugate foci, to the reflector.

The object at O is generally ſome ludicrous or frightful figure, on purpoſe to divert the ſpectators, it is painted upon a piece of plain thin glaſs, in dilute and tranſparent colours; for if they were opake they could not be repreſented at the focus L. They are beſt done with oil of ſpike, mixt with ſeveral ſorts of colours; they penetrate the glaſs and dry preſently. The pieces of glaſs the figures are painted on, are put into ſliders, three of them in one ſlider; which are to be put in at a ſlit at O in the tube HI, when uſed, and in an inverted poſition; they muſt be placed in a line between E and D. Small living animals may alſo be uſed; and ſome of them make a moſt terrible appearance.

This machine carried into a dark room, and ſet on one ſide, and the candle or lamp E being lighted, and the box made cloſe, the candle partly by its direct light, but more by that reflected from the ſpeculum GN, will ſtrongly illuminate the object at O, which being tranſparent, the light will paſs thro' it, and being refracted by the lens CD to the oppoſite wall, will give there the image erect, becauſe the object was inverted. If the image at L is not diſtinct; then to adjuſt it, draw the tube CD a little in or out, till the image appears diſtinct,

tinct, for it may be projected to any diſtance. As Fig.
there is a deal of ſmoak produced within the lan- 128.
thorn, there is a hole R at the top, to convey it away.

Inſtead of the concave reflector GN, one may place a deep convex lens AB, in the end of the tube HI, which will throw the light of the candle upon the object at O, and illuminate it. Likewiſe one may place two lenſes in the tube CK, which will make the rays converge ſooner; and then there muſt be placed a ſtop between them with a hole to let the light thro', and cut off the ſuperfluous rays. The whole length of the machine may be about a foot and a half long; and the focus of the glaſs CD, ſix or eight inches. This inſtrument and the ſolar microſcope are both upon the ſame theory.

PROB. XXIV.

To make a Camera Obſcura for taking the draught of a country, a town, or any building, &c.

Let ABCD be a wooden box, about a foot and 129.
a half ſquare, open only on the ſide AD; and *efgh* a ſmaller box about ſix inches ſquare, and open only on the under ſide *gh*. This leſſer box is let into a hole in the top of the great box AB, and made cloſe and fixt there; *eopi* is a plain reflector or looking-glaſs, fixt diagonally in the ſmall box, with the reflecting ſide downwards, making an angle of 45 degrees with the perpendicular ſide, or face, *eh*. There is a tube K going thro' the ſide *eh* of the ſmall box, which ſlides in and out; in this tube is contained a convex lens about three inches diameter, and focal diſtance about two feet; this tube ſtands even with the middle of the reflector *ep*. The focal diſtance of the lens ought to be equal to the height of the middle of the reflector

Fig. 129. above the bottom CD of the great box, added to the diſtance of the ſaid middle point from the lens. On the top of the ſide AD, along AL there hangs a curtain; by the help of this, the box may be made entirely dark within.

When this machine is to be uſed, it is carried into the fields, to the top of a hill, or ſome high piece of ground, where the place, whoſe draught is to be taken, may be ſeen to the beſt advantage. Then placing it upon a table, or upon a frame made on purpoſe; ſet it level upon its baſe, and let it exactly face the objects that are to be drawn. Then the draught's man is to put his head and his hands into the open ſide AD, and draw the curtain cloſe about him, that no light get in; then laying a ſheet of paper upon the bottom CD, and faſtening it that it cannot move; ſee if the objects appear diſtinct upon the paper; if they do not, move the tube a little inward or outward, till the objects do appear diſtinctly; and then it is time to draw the pictures of the objects within. Therefore having your head and hand in the box, and the curtain cloſe drawn, run over all the lines of the ſeveral objects, as they appear on the paper, with a black lead pencil, or one of chark; till you have gone thro' the whole. Then you may either keep the draught as it is, or run the lines over again with ink, and then it will ſtand in no danger of being rubbed out. Theſe inſtruments generally take in about 45 degrees.

This machine is on the ſame principle as that of darkening a room, and letting the light in thro' a lens in the window ſhut. For here the rays flowing from any object, are refracted by the lens in the tube K, and would paint the image of that object, in its focus; but being reflected by the ſpeculum *ep*, directly down to the baſe DC, they will there paint the image thereof, at the ſame diſtance from the reflector, as it would have been beyond it, in their true ſituation, and proper colours. Here the lens muſt not be too broad, for then the images will not be diſtinct; nor ſhould it be too narrow, for then they will appear dark. And of all things a clear ſunſhiny day muſt be choſen for this work, ſo as to ſhine on that ſide of the objects that appears to your view. Fig. 129.

Inſtead of having a tube with the lens in the ſide of the box; the tube may be fixt perpendicular under the reflector, and then the rays will be firſt reflected, and then refracted. This requires a greater height of the box; but all the effects will be the ſame.

Alſo inſtead of having the ſide AD open, one may have it cloſe, and near the bottom make a hole to put the hand thro' with a pencil; and near the top make another hole to look thro; and there the ſide of the box ſhould be ſlanted off. The part where the hand goes thro', ſhould ſlide back and forward, that the hand may move to any part of the paper.

PROB. XXV.

To make a portable Camera Obſcura, to view diſtant objects.

130. ABCD is a ſquare box eight inches long, open at top and on the ſide AD. ALMD a ſquare hollow fruſtum of a pyramid five inches long, fixed cloſe to the ſide AD of the box. In the end of this pyramid LM, there ſlides a tube KI, in which is fixt a broad convex lens, whoſe focus is near the end BC. FG is a plain reflector or looking-glaſs going diagonally thro' the box, and is ſet to an angle of 45 degrees. EB is a glaſs plane, having the upper ſide unpoliſhed; AH is a lid that opens up when the machine is uſed; this lid has two wings,

Fig. 130. on each ſide, hanging perpendicularly, and they reſt upon the ſides of the box when the lid is up; and ſerve to ſhade the image, made in the rough glaſs EB, from the light.

When this inſtrument is uſed, the lid is opened, and the ſide AD ſet towards the object. Then the rays coming from thence, are refracted by the lens in the tube KI, and would form the image upon the ſide BC; but being reflected upwards by the looking-glaſs FG, they form the image upon the glaſs plane EB, with the right end uppermoſt, when one faces the objects; but the left ſide is to the right, and the right to the left; where it may be ſeen diſtinctly with the eyes. If the image does not appear diſtinct, draw the tube KI out or in, gradually, till it does appear diſtinct. Then the image may be traced out on the glaſs with a black lead pencil. This operation is beſt performed, when there is a ſtrong ſunſhine upon the objects.

Or thus,

131. ABCED is a cheſt in form of a long parallelopided, nine inches broad, and two feet long; in the end AC is a tube, with two convex glaſſes F, G; this tube may be drawn out or put in, at pleaſure. H is the focus of the lens F, and FH its focal diſtance; alſo HG is ſomething leſs than twice the focal diſtance of the lens G. The lenſes F, G ought to be pretty broad, the diameter of F about $\frac{6}{10}$; of G, $\frac{4}{10}$ of a foot. IKLM is an oiled paper, or a plane glaſs, parallel to the end DE, and whoſe diſtance from the lens G is ſomething more than twice its focal diſtance, and from the end DE, ſix or eight inches. N is a ſlit big enough to look thro' with both eyes. The whole cheſt is cloſe and dark within; no light entering, but what comes thro' the tube. To make the rays converge ſooner, three lenſes may be uſed.

Then the tube being directed to any objects, the rays of light proceeding from them, will be refracted by the lens F to its focus H, where the images will be made, and from thence paſſing thro' the lens G, will be refracted again to their focus upon the paper IKLM, being the conjugate focus to H; and therefore a perſon looking in at the ſlit N, will perceive them painted upon the paper in their true and natural poſition. If the images appear not diſtinct, draw the tube FG, in or out, till they are ſo. Fig. 131.

SCHOLIUM.

The pictures of external objects may be made in a dark room, by holding a concave ſpeculum directly before a ſmall hole in the window ſhut, at a diſtance equal to the focal diſtance. For the ſpeculum will paint the images upon the window ſhut, or a paper held there, but inverted.

Or the pictures of objects may be made in a dark room with a convex lens placed in the window ſhut, or in a ſky-optic ball which goes into a hole in the window ſhut; but then the images will be inverted. But receiving them upon a white apron, and looking down at them, they appear erect.

But to have them direct, hold a plain reflector, ſlanting from the lower ſide of the hole, upwards, to reflect the light upwards; and ſtanding with your back to the window, you'll ſee the images upon the cieling, or on a paper held at a proper diſtance above the reflector, in their proper ſituation. This is like fig. 130, in this Problem.

Or thus,

Place the reflector in a contrary poſition; that is, cloſe to the lens on the upper ſide, and ſlanting downwards; and it will project the images upon a horizontal paper near the floor, in their proper ſituation; which may be ſeen, by ſtanding

Fig. with the back towards the window, for this has
131 the ſame effect, as fig. 129, in Prob. 24.

Theſe two laſt methods are preferable in this, that they repreſent objects in their natural ſituation. But they have this diſadvantage, that they want light. Therefore one muſt have very good glaſſes, a dark room, and a ſtrong ſunſhine on the objects.

PROB. XXVI.

To make a machine to magnify a perſpective draught.

132. Here ABCDE is a wooden box eight inches ſquare, open only at the bottom DI, L is a convex lens three or four inches diameter, and about two feet focal diſtance; this is placed in the foreſide of the box CD; alſo CIEF is a plane reflector, ſet at an angle of 45 degrees, and with the reflecting ſide downwards; G, G, G, G, are the legs, the inſtrument ſtands on, whoſe height muſt be ſuch, that the diſtance from the lens to the middle of the reflector, and thence to the baſe, muſt be equal to the focal diſtance of the lens; P is the draught to be viewed, laid on the baſe.

When you would view any draught or piece of perſpective, lay it flat on the baſe at P, with the upper end towards you, and looking into the box, thro' the lens L, you will ſee it diſtinctly, and magnified.

For the rays iſſuing from the draught at P, being placed in the focus, and reflected to the lens, and then emerging parallel to the eyes, will repreſent the objects vaſtly diſtant, and conſequently as much magnified; having the very ſame appearance as the natural objects themſelves would have. Only the right hand is to the left, and the left to the right.

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Fig. 131 with the back towards the window, for this ha the ſame effect, as fig. 129, in Prob. 24.

Theſe two laſt methods are preferable in this that they repreſent objects in their natural ſituation But they have this diſadvantage, that they wan light. Therefore one muſt have very good glaſſes a dark room, and a ſtrong ſunſhine on the objects.

PROB. XXVI.

To make a machine to magnify a perſpective draught.

132. Here ABCDE is a wooden box eight inches ſquare, open only at the bottom DI, L is a convex lens three or four inches diameter, and about two feet focal diſtance; this is placed in the foreſide of the box CD; alſo CIEF is a plane reflector, ſet at an angle of 45 degrees, and with the reflecting ſide downwards; G, G, G, G, are the legs, the inſtrument ſtands on, whoſe height muſt be ſuch, that the diſtance from the lens to the middle of the reflector, and thence to the baſe, muſt be equal to the focal diſtance of the lens; P is the draught to be viewed, laid on the baſe.

When you would view any draught or piece of perſpective, lay it flat on the baſe at P, with the upper end towards you, and looking into the box, thro' the lens L, you will ſee it diſtinctly, and magnified.

For the rays iſſuing from the draught at P, being placed in the focus, and reflected to the lens, and then emerging parallel to the eyes, will repreſent the objects vaſtly diſtant, and conſequently as much magnified; having the very ſame appearance as the natural objects themſelves would have. Only the right hand is to the left, and the left to the right.

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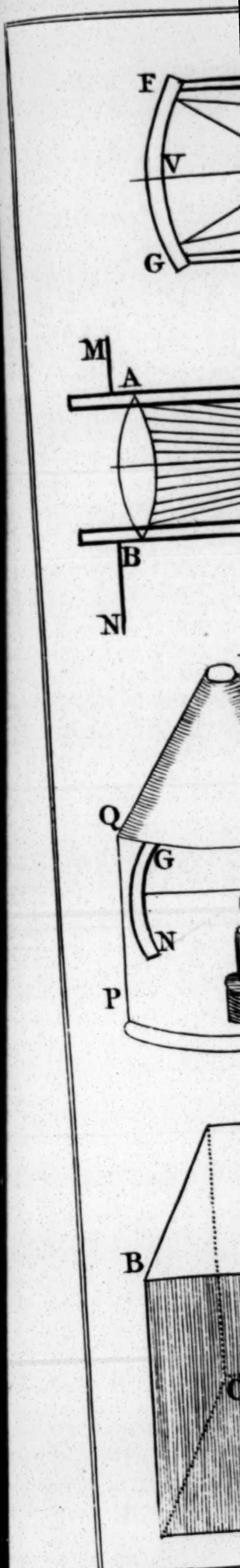

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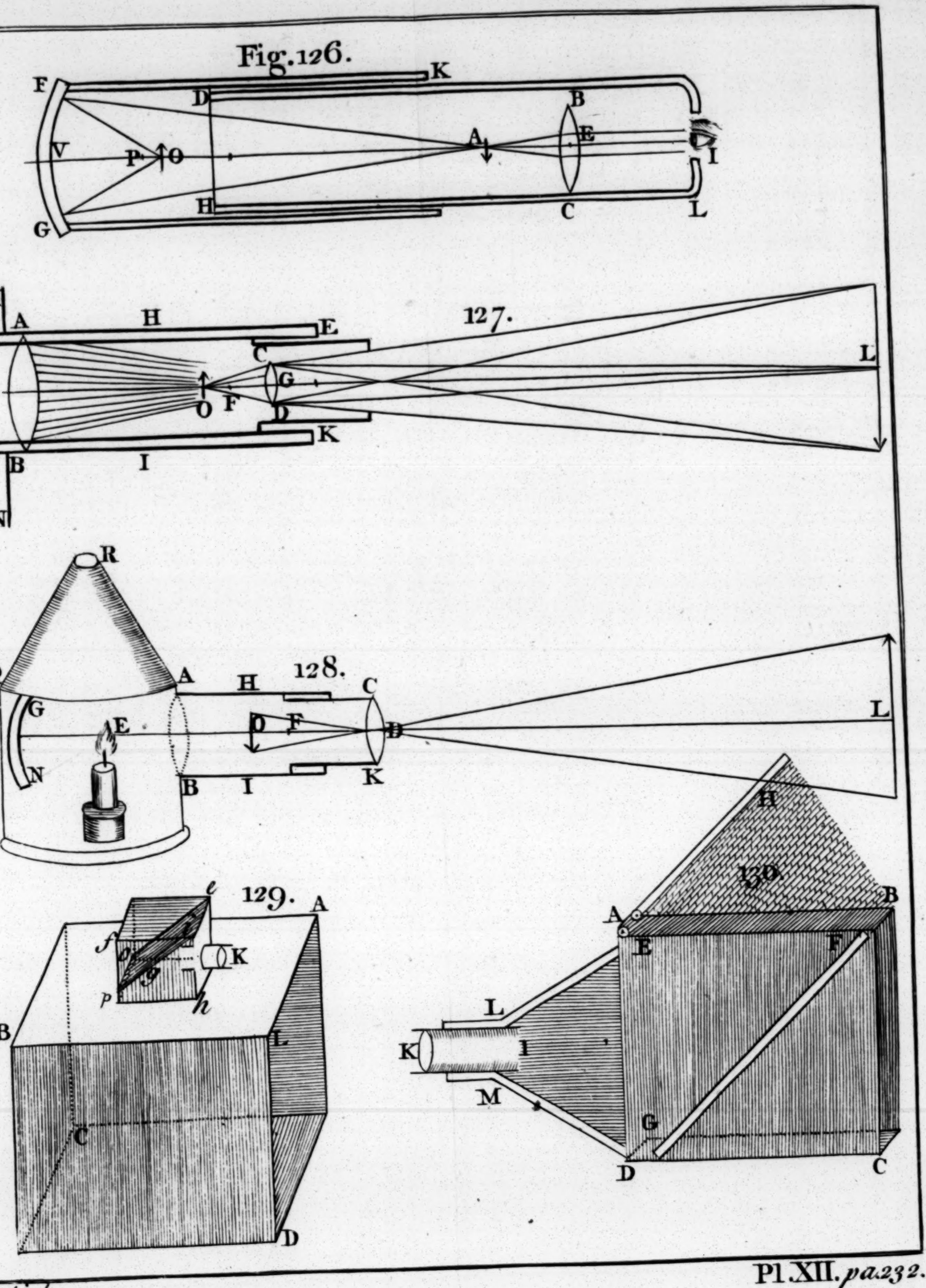

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Pl. XII. pa. 232.

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For as the draught is turned upſide down, it will appear contrary in the machine; that is, it will appear the right way up. If it was laid the right way up, the right ſide would appear to the right; but as it is wrong way up, the right ſide appears to the left. And therefore one cannot read any writing this way, unleſs one can read it backward. Fig. 132.

This machine is the ſame as that in fig. 129, taking away the box ABD. For as the external objects give the picture thereof on this baſe CD; ſo on the contrary the picture placed there, would ſend the rays back again to the ſeveral objects; but the eye takes them before they come there, and they paint their images upon the retina; but upſide down, becauſe the face is turned the contrary way.

But inſtead of legs, one may have it all cloſe below, the better to conceal it; as BH, which is a hollow box, cloſe below, but above having the ſame apparatus as the laſt; at the bottom a dozen views are laid flat upon one another at P; which are all ſhewn ſucceſſively, and then drawn back, by the rulers O, into the part GK behind. EG a thin cloth to let the light thro'; which is ſhut by the ſlider BE, when the inſtrument is not uſed. There may be two or three lenſes L, on the ſide CH, for different people to look thro' at once; which are ſhut up by ſliders when not uſed. 133.

Otherwiſe thus.

ABE is a wooden ~~box~~ open on the top, the ſide AC a foot long, or more, and almoſt as broad; D a concave reflector, 9, 10 or 12 inches broad, fixt perp. with its back leaning a little againſt the ſide BCK; its focal diſtance 12 inches, equal to the length AC. HG a ſlit in the end AE to look thro'. EH, FG two ſtops or bearers, faſtened to the oppoſite 134.

Fig. 134. poſite ſides of the box, againſt theſe the picture or draught is to reſt.

When a piece of perſpective is to be viewed, you muſt ſlide it down upon the pieces EH, FG, as low as the ſlit, and with the right end upwards; and facing the reflector; and then looking in at the ſlit HG, it will appear magnified and erect, but the right hand will be to the left, and the left to the right.

For ſince the print leans with the upper end towards the reflector, and the reflector leans a little back, the rays iſſuing from the picture will not be reflected back upon the picture, but will paſs below, thro' the ſlit HG, to the eye; and the picture being in the focus of the reflector, the rays are reflected parallel upon the eye, and therefore it will ſee the picture diſtinctly, and very much magnified.

Or thus.

But the beſt way of all is, to place the view upon a table, to ſtand perpendicular againſt a wall, or hang it on a pin; and then to place a lens at its focal diſtance from it; then looking thro' the lens, you will ſee the proſpect in its natural poſition every way; and magnified ſo as to appear like the natural place itſelf; eſpecially if the view be painted in its proper colours, as it ought to be.

To have the proſpect to the beſt advantage, the lens ſhould not be leſs than four inches diameter; and two feet, or two feet and two or three inches focal diſtance. And in order to confine the view to your proſpect, the lens ſhould be ſet upright in the middle of a little box or tube ſix or eight inches long; at the end of which, a ſlit muſt be made an inch wide, to look thro' with both eyes, towards the object; and therefore this box muſt be placed horizontally to face it. The picture ſhould be ſo large, as to ſubtend an angle of 25 or 30 degrees, at the focal diſtance of the glaſs. But when your picture is given, you muſt ſuit it with a glaſs, ſo that the focal diſtance of it, may be about twice the breadth of the picture. A glaſs ſhould never be of ſo ſmall a focal diſtance, that the ſcratches and deformities of the picture may appear. For at too ſhort a diſtance the glaſs will magnify theſe errors and make them viſible, and ſpoil the proſpect. The way to judge of a proper glaſs for any picture, is this; obſerve at what diſtance the picture makes the beſt and fineſt appearance to the naked eye; and that ought to be the focal diſtance of the glaſs. So that altho' a picture placed in the focus of any lens, will be magnified and ſeem at a vaſt diſtance; yet to prevent any blemiſhes or irregularities in the drawing to appear, the lens ought to be choſen by the former rule. A ſhort glaſs will not take in all the view; and a long glaſs makes the picture have but a ſmall appearance; and the eye cannot diſtinguiſh the minute parts.

I ſhall further obſerve, that to have any perſpective view appear exactly ſimilar to the place it was drawn for; the lens ought to be placed in the point of view in the air, of that picture or perſpective draught; and conſequently the focal diſtance of the lens muſt be equal to the principal ray (or line of diſtance); and when this is ſo, if it be viewed thro' that glaſs, it will give the true appearance of that place in all reſpects, in regard to the magnitude, and ſituation of the ſeveral parts. And therefore all theſe perſpective draughts ought to be ſo drawn, that the point of view be at the due diſtance, that is, equal to the focal diſtance of the glaſs proper to be uſed. The longer the focal diſtance of the glaſs is, the longer and more extended, the place appears to be, whoſe perſpective you view. And the ſhorter the glaſs, the ſhorter and more contracted is the apparent view. Both theſe

Fig. theſe diſtort the view, the longer glaſs by making it too long; and the ſhorter by making it too ſhort, but they do not affect the breadth. And nothing but a glaſs of a juſt length will make it appear in its juſt proportion. If a proſpective draught be well taken, and then viewed according to theſe rules, you'll have the exact appearance of the place, juſt the ſame as if you was looking at it with your naked eyes.

It may be thought, that inſtead of a lens, two ſmall glaſſes of the ſame focal diſtance, for the two eyes, may do as well. But theſe will not anſwer the ſame end, as one large glaſs; becauſe the axes of the two pencils of rays, that fall on theſe two glaſſes, interſect at the picture, and no further off, and there make an angle; and ſo enter the eye; whereas they ought to be parallel. And this makes the picture not appear far enough off, and quite ſpoils the view.

Many other optical inſtruments might be inſerted, but theſe that have been here explained, are the principal; and from theſe, all others of a like nature are eaſily underſtood, and their effects demonſtrated by a like way of reaſoning.

Likewiſe by the known properties of light, the phænomena of all ſorts of meteors may be explained, and the reaſon of their colours ſhewn; ſuch as the Rainbow, Corona's, Halo's, Parhelia's, Paraſelene's, &c. But I ſhall here only explain the colours and appearances of the rainbow, becauſe it is a phænomenon ſo common, that it comes under every body's obſervation.

PROB.

Fig.

PROB. XXVI.

To explain the magnitude, poſition, and colours of the Rainbow.

The rainbow never appears but when it rains in the ſun-ſhine; and is cauſed by the refraction and reflection of the ſun's light paſſing thro' the drops of falling rain. And it may be made artificially, by ſpouting up water, and cauſing it to fall down in drops, in the ſun's rays; in theſe drops the colours will appear to a ſpectator ſtanding in a due poſition. And the like may be ſhewn in globes of glaſs filled with water, and placed in the ſun in a proper poſition.

To underſtand how the bow is made; let BNFG 135.
be a drop of rain of a ſpherical figure, C its center, and let AN be one of the ſun's rays incident upon it at N, and then refracted to F, where let it either go out of the drop by refraction to V, or be reflected to G; and at G let it either go out by refraction to R, or be reflected to H; and at H let it either go out by refraction to S, or be reflected to I, &c. On the incident ray AX, and the refracted one NF, let fall the perpendiculars CD, CE. Draw the diameter BQ parallel to the incident ray AN. And let *m*, *n* be the ſines of incidence and refraction out of air into water. Draw NC, CF.

Now in the progreſs of the ray AN, thro' the points N, F, G, H, &c. it is continually changeing its direction, the ſame way about the circle, till at laſt it emerges out of the drop, making the angle of refraction, equal to that at its incidence; for all the lines NF, FG, GH &c. are equal. Then to find the direction in which it at laſt emerges, produce NF to W, FG to T, GH to Y, &c;

Fig. 135. &c; and then we muſt find the ſum of the angles XNW, WFT, TGY &c. and YHS, which makes the whole change of direction, or the revolution of the ray in its progreſs. Draw CZ perp. to FG. Then in the quadrilateral CEFZ, the angles at E and Z are right, therefore the angle $ECZ + EFZ, = $ two right angles $= EFZ + ZFW$, therefore $ECZ = ZFW$, or $ZFW = NCF = 2NCE$. Put $t =$ number of reflections, F, G, H, &c. Then ſince all the external angles at F, G, H are equal, the ſum of them all is $= t \times ZFW = 2t \times NCE$, to which add $XNF + YHS$ or $2XNF$, and the ſum is $2t \times NCE + 2XNF$, for the whole change of direction of the ray. But $XNF = XNC - ENC$, and $NCE = R - CNE$ (putting $R =$ a right angle), therefore the whole change of direction is $= 2tR - \overline{2t + 2} \times CNE + 2CNX$, or twice the quantity $tR - \overline{t + 1} \times CNE + CNX$. And this being known, it is eaſy to find the angle the emerging ray makes with the incident ray AN or BQ. This is true of any ray whatſoever, but to our purpoſe there is but one poſition of the incident ray AN, which will bring the colours, and this is next to be found.

Let AN be an incident ray of any colour, and let it emerge at H, deſcribing the tract ANFGHS. Now that the rays of any colour emerging at H, may affect the ſenſe ſtrongly, they ought to emerge parallel, for if they emerge and fall upon the eye, in different directions, they mix with other rays of a different colour, and cannot appear ſenſible. And there will be ſo few rays of that colour in any one direction, that they cannot excite any ſenſation. And ſince to excite a ſtrong ſenſation, the rays of any one colour ought to emerge parallel, and as they are alſo parallel at their incidence; therefore we muſt find what poſition, or what angle of incidence, the ray AN muſt have, that all the rays of that colour, emerging at a ſmall diſtance from H, may Fig. 135. be all parallel to HS. Now this can only be when the ſum of all the angles $2tR - \overline{2t + 2} \times CNE + 2CNX$ is a maximum or minimum. For by the nature of a maximum or minimum, ſuch a quantity remains the ſame for a ſmall variation of the arch BN. Now we may obſerve, that when the arch BN is very ſmall, the arch FQ is very ſmall; and as BN grows greater, QF grows greater, and BF leſſer; and when BN approaches a quadrant, QF grows leſs again, and BF bigger. So that BF firſt decreaſes to a certain limit, and then increaſes again, and at that limit it is at its minimum. But the quantity $2tR - \overline{2t + 2} \times CNE + 2CNX$ depends much upon the ſituation of F, and therefore will be at its minimum when the rays at H emerge parallel. Now ſince $2tR$ is given, $2tR - \overline{2t + 2} \times CNE + 2CNX$, will be at is minimum when $\overline{2t + 2} \times CNE - 2CNX$ is a maximum, or when $\overline{t + 1} \times CNE - CNX$ is a maximum. But (by Prob. 83. Sect. VII. Algebra) $\overline{t + 1} \times CNE - CNX$ is a maximum, when $\overline{t + 1} \times$ increment of CNE = increment of CNX.

Let $y = CD$ or NM the ſine of incidence; $CN = 1$; m, n the ſines of incidence and refraction, and take Nn a very ſmall increment or decrement of BN, and drawing no parallel to AN, No will be the increment or decrement of MN; then $\frac{n}{m} y = CE$ the ſine of refraction, and its increment $= \frac{n}{m} \times No$. And by the nature of the circle, coſine $(\sqrt{1 - yy})$: radius (1) :: $No : Nn = \frac{No}{\sqrt{1 - yy}}$, the increment of BN or $\angle$ CND. Alſo coſine $\left(\sqrt{1 - \frac{nnyy}{mm}}\right)$: radius

Fig. 135. dius (1) : : increment of the ſine of refraction $\left(\frac{n}{m} \times No\right) : \frac{n \times No}{m\sqrt{1 - \frac{nnyy}{mm}}}$, the increment of the arch or angle of refraction CNE. Therefore $\overline{t+1} \times \frac{n \times No}{m\sqrt{1 - \frac{nnyy}{mm}}} = \frac{No}{\sqrt{1-yy}}$. Put $p = t + 1$, then $\frac{pn}{\sqrt{mm - nnyy}} = \frac{1}{\sqrt{1-yy}}$, and $pn \times \sqrt{1-yy} = \sqrt{mm - nnyy}$; and ſquaring, $ppnn - ppnnyy = mm - nnyy$; and reducing, $ppnnyy - nnyy = ppnn - mm$; and $y = \sqrt{\frac{ppnn - mm}{pp - 1 . nn}}$. And y being known, the angles CND and CNE are known, and conſequently the quantity of the angle of deviation of the ray, $2t\mathrm{R} - p \times \mathrm{CNE} + 2\mathrm{CNX}$. And the angle in which HS cuts the incident ray AN or BQ, will be known; for if the deviation be 180 degrees, the incident and emergent rays will coincide, and the emergent ray will return back to the ſun.

Since y depends upon the quantities of m and n, it is evident that the rays of different colours, having a different degree of refrangibility, will have different limits of BN, and will emerge parallel in different points H, and will cut BQ at different angles. And theſe ſeveral ſorts of rays being ſeparated from one another, will appear each in their proper colours.

For the firſt rainbow; if light paſſes out of air into water, m and n (by Schol. Prop. II. B. III.) will be 108 and 81 for the leaſt refrangible rays, or the red; and 109 and 81 for the moſt refrangible rays, or the violet. And therefore in this bow, where $m = 108$, $p = 2$, y comes out $=$ 86036 = S. 59 22 = CNX, and CNE = 40 11. And 2R — 4 CNE + 2CNX = 137 58, and this

or its ſupplement 42 2 is the angle the emergent Fig.
ray makes with the incident ray for the red. And 155.
in like manner when $m = 109$, the angle which the emergent ray makes with the incident ray, will be found 40 17, for the violet.

Again in the ſecond bow where $p = 3$, $m = 108$, we ſhall find $y = 95015 = \text{S. } 71\ 50 = CNX$, and $CNE = 45\ 27$; and $4R - 6CNE + 2CNX = 230\ 57$ which exceeds 180 by 50 57, the angle made with the incident ray, for the red. And when $m = 109$, the angle the violet emergent ray makes with the incident ray will be 54 7.

Now ſuppoſe O to be the eye of the ſpectator, 156.
and OP a line drawn thro' the ſun and the eye. And let the angles POE = 40 17, POF = 42 2, POG = 50 57, and POH = 54 7. And let theſe angles be turned about their common ſide OP; then ſhall the other ſides OE, OF, OG, OH, deſcribe the extreme arches of two rainbows AFBE, and CHDG. For if E, F, G, H be drops of water placed any where in the conical ſurfaces deſcribed by OE, OF, OG, OH, and be illuminated by the ſun's rays SE, SF, SG, SH; the angle SEO or POE of 40 17 will be the greateſt angle in which the violet rays will after one reflexion be refracted to the eye; and therefore all the drops in the line OE, ſhall ſend the moſt refrangible rays moſt copiouſly to the eye, and thereby ſtrike the ſenſes with the deepeſt violet colour. And in like manner the angle SFO or POF of 42 2, will be the greateſt angle under which the red rays can emerge out of the drops, and therefore will come moſt copiouſly to the eye and ſtrike the ſenſes with that colour. And the rays that have intermediate degrees of refrangibility, ſhall come moſt copiouſly from drops between E and F, and ſtrike the ſenſes with the intermediate colours; that is, from the inſide to the outſide of the bow, the colours will

Fig. appear in the order violet, indigo, blue, green,
135. yellow, orange, red.

Again in the ſecond bow, the angle SGO or POG of 50 57 ſhall be the leaſt angle under which the red rays, after two reflexions can emerge out of the drops, and come moſt copiouſly in the line OG, and ſtrike the ſenſe with that colour. And the angle SHO or POH of 54 7 ſhall be the leaſt angle in which the violet rays after two reflexions, can emerge out of the drops, and therefore they will come moſt copiouſly to the eye; and ſtrike the ſenſe with that colour. And by the ſame argument the drops between G and H, ſhall ſtrike the ſenſe with the intermediate colours; therefore from the inſide to the outſide of the bow, the colours will appear in this order, red, orange, yellow, green, blue, indigo, violet. And ſince theſe four lines OE, OF, OG, OH, may be ſituated any where in the conical ſuperficies, what is ſaid of the drops and colours in theſe lines, is to be underſtood of the drops and colours every where in thoſe ſurfaces. But the ſecond bow, by reaſon of the light loſt by two reflexions does not always appear, and is always fainter than the firſt.

Theſe would be the angles if the ſun was but a point, but from his apparent diameter, the breadth of the bows will be increaſed half a degree, whence the breadth of the inner bow will be 2 15, and of the outer one 3 40; and their diſtance 8 25; the radii of the inner bow 40 2 and 42 17, of the outer one 50 42 and 54 22; and this appears to be ſo by obſervation, when their colours are ſtrong and lively.

136. It may be obſerved, that all the rays but the violet in the line SE, will emerge from E in a greater angle than SEO made by the violet, and will therefore all paſs below the eye: and all the rays but the red, in the line SF, will emerge from F in a leſſer angle than SFO made by the red, and will therefore

paſs above the eye; by which means, only red will appear in the line OF, and only violet in the line OE. Fig. 136.

They that have a mind to meaſure the bow, may obſerve that its radius is equal to the height of the bow and the height of the ſun together; and therefore is eaſily meaſured with a quadrant. And if the ſun's altitude be greater than 42 17, the inner bow cannot be ſeen; nor the exterior one, if his altitude exceed 54 22.

In the third and fourth rainbows y is .97373 and .98366 reſpectively, and CNX $=$ 76 50 and 79 38. Whence $2t\mathrm{R}+2\mathrm{CNX}-2p\times\mathrm{CNE}=$ 318 20, and 400 31 reſpectively, putting m and n as 4 to 3 for the red rays. And therefore in the 3d rainbow, the emerging ray cuts the incident ray at an angle of 41 40; and in the 4th, at an angle of 40 31, both moving from the ſun. And therefore looking towards the ſun, the red rays in theſe rainbows would appear at the diſtance from him of 41 40 and 40 31 reſpectively, and nearly coinciding. And in the 3d the violet would be towards the ſun; but in the fourth the violet is from the ſun. But their light is weakened ſo by reflexions, that they never appear. 135.

Cor. 1. *The angle of revolution* $2t\mathrm{R}+2\mathrm{CNX}-2p\times\mathrm{CNE}$ *will come to its limit when* ND *is to* NC :: *as* $\sqrt{mm-nn}$ *to* $\sqrt{\overline{pp-1}\times nn}$. *And* ND *is to* NE :: *as* m *to* pn.

For $\mathrm{ND}^2=1-yy=1-\dfrac{ppnn-mm}{\overline{pp-1}\,.\,nn}=\dfrac{mm-nn}{\overline{pp-1}\,.\,nn}$. And $\mathrm{NE}^2=1-\dfrac{nn}{mm}yy=1-\dfrac{ppnn-mm}{\overline{pp-1}\,.\,mm}=\dfrac{ppmm-ppnn}{\overline{pp-1}\,.\,mm}$. Therefore ND : NC :: $\sqrt{\dfrac{mm-nn}{\overline{pp-1}\,.\,nn}}$: 1 :: $\sqrt{mm-nn}:\sqrt{\overline{pp-1}\,.\,nn}$. Alſo ND : NE :: $\sqrt{\dfrac{mm-nn}{\overline{pp-1}\,.\,nn}}:\sqrt{\dfrac{ppmm-ppnn}{\overline{pp-1}\,.\,mm}}::\sqrt{\dfrac{1}{nn}}:\sqrt{\dfrac{pp}{mm}}::\dfrac{1}{n}:\dfrac{p}{m}::m:pn$.

Cor.

Fig. 136. Cor. 2. *When the angle of revolution (or change direction) of the emerging ray differs not much fro a ſemicircle or any odd number of ſemicircles; one mu look for the bow towards the ſun's oppoſition. Bu when it differs not much from a circle or any numbe of circles, one muſt look towards the ſun for the bow Thus the firſt and ſecond bows are ſeen oppoſite to th ſun, and the third and fourth towards it.*

For if the revolution be a ſemicircle, or 3, 5, 7 &c. ſemicircles, the direction of the ray is toward the ſun; and therefore, that it may enter the eye one muſt look the contrary way. But if the revolution be 1, 2, 3 &c. whole circles, the ray move directly from the ſun, and therefore to receive i into the eye, one muſt look towards the ſun; fo the face muſt be ſet againſt the motion of the ray.

Cor. 3. *If the angle of revolution falls ſhort of a ſemicircle or any number of ſemicircles, the violet colour will be on the inſide the bow. But if it exceed them, the violet will be on the outſide. Thus in th firſt and third bows, the violet is within; and in th ſecond and fourth, without.*

For the violet rays by a greater refraction, make a greater revolution than the red; and therefore when the revolution angle falls ſhort, the violet ray will come nearer a ſemicircle, and make a leſs angle with the incident ray; and therefore will be neareſt the center of the bow. But the contrary will happen when the revolution is greater than a ſemicircle, for then the violet recedes farther off, and makes a greater angle with the incident ray; and ſo it will be furtheſt from the center of the bow. Thus in the firſt bow the violet is within, in the ſecond without, in the third within, and in the fourth without. But after 3 or 4 reflexions the light is ſo weak as not to affect the ſenſes; and ſo the third and fourth bows, and thoſe that follow, never appear; and therefore I ſhall ſay no more about them.

FINIS.

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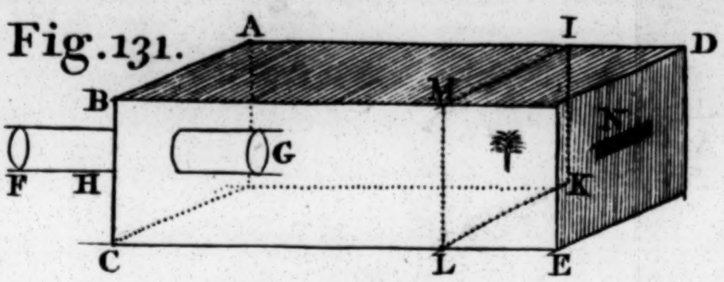
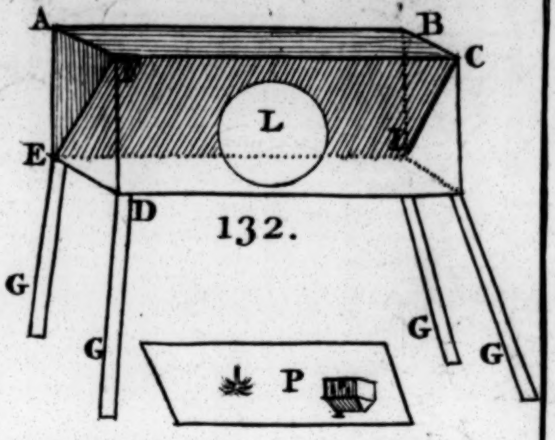
Fig. 131.

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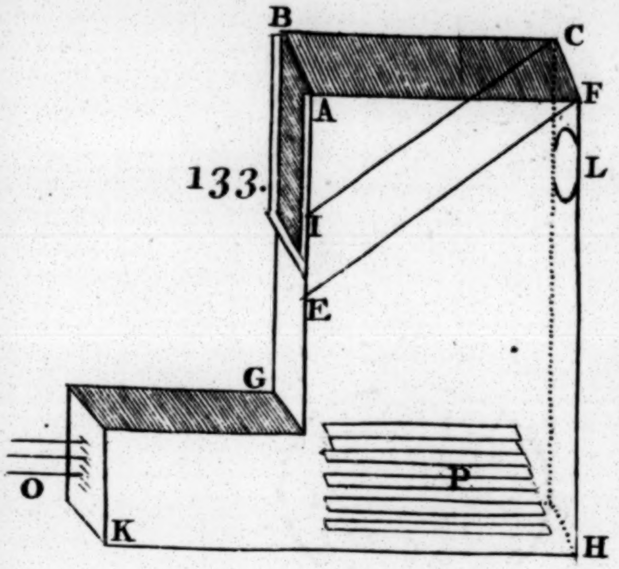
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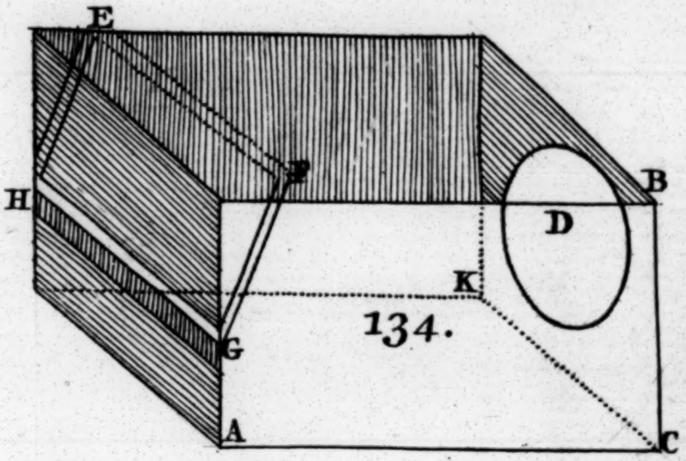
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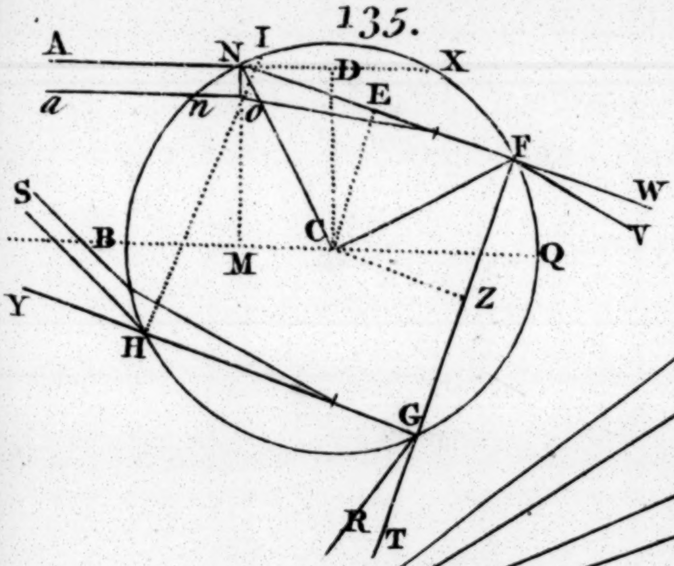
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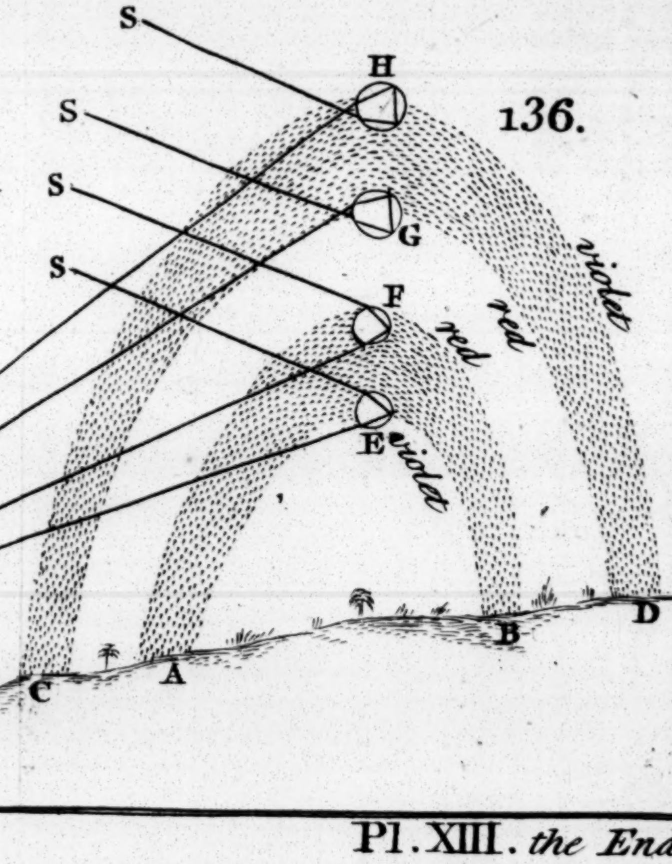
136.

Opticks.

Pl. XIII. *the End.*

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ERRATA in the OPTICS.

page	line	read
85	5	$\dfrac{Cfxx}{\dfrac{rr}{2Cf}-x}$.
169	14	$\left(\dfrac{q-s}{q+s}\right)^2 t$.
173	15	thro' the lens B,
220	25	microſcope, which is ſhorter. **Where**
224	29	focus of AB.

ERRATA in the PERSPECTIVE.

In the Plates.

Fig.	
19	For *t* put *l*.
57	The pricked lines 41, 1S, and ſome others wanting.

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